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Dark Energy Cosmological Model With Constant Deceleration Parameter in f(T) Theory

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Abstract: *The Bianchi type-I dark energy model in the $f(T)$ theory of gravity has been examined in this study. We have examined Bianchi type-I solutions employing constant deceleration parameter. Some physical properties of the examined model are also covered in details.*
Keywords: *$f(T)$ Theory, Dark Energy, Bianchi Type-I Universe, Constant Deceleration Parameter.*

I. INTRODUCTION

Theory of Relativity by Albert Einstein, which was proposed at the beginning of the 19th century, is regarded as an innovation to modern cosmology. Later, Riemann space time developed this theory in accordance with the Levi-Civita connection, a torsion-free and metrically compatible link that helps us grasp the universe's underlying geological structure. The challenges encountered by the aforementioned hypothesis include initial singularity, cosmological constant, fine tuning, cosmic co-incidence, and flatness [1,2]. Einstein's theory of gravitation had a significant impact on the development of a model of cosmology and the explanation in terms of the Universe's creation and evolution. However, late-time acceleration could not be explained by Einstein's theory, which is one among the most important topics in contemporary cosmology [3]. To explain how dark energy and dark matter, as well as late-time acceleration, exist in the universe, a number of modified gravity theories, such as those that have recently been proposed, have been put forth [4]. The Wilkinson Microwave Anisotropy Probe (WMAP) finds indirect evidence explain the Universe's accelerated expansion in the Cosmic Microwave Background (CMB) radiation [5,6] and large-scale structure [7].

According to Einstein's theory, several researchers are making significant attempts to observe dark energy permeated the entire universe. The Universe's fast expansion is caused by a type of negative pressure force referred described as dark energy [8]. To understand the character of the Universe's accelerated expansion, we have numerous theoretical models of dark energy to choose from, including the quintessence scalar field models [9,10], the phantom field [11-13], K-essence [14,15], tachyon field [16], quinton [17,18], and Chaplygin gas [19,20]. Approximately 74% is the dark energy of the Universe, dark matter is 22% and the remaining 4% is regular matter [21-24].

Among the modified theories of gravity are $f(R), f(R,T), f(G), f(G,T)$ and $f(T)$. The modified theories of gravity have the significant benefit of explaining the cosmic acceleration in late time and early time expansion. Among these are ideas, the generalized teleparallel theory of gravity attracted attention due to its elucidation of dark energy. In 1928, Einstein suggested teleparallelism to combine gravity and electromagnetic into a unified field theory in which space-time is incorporated a connection with diminishing curvature but nonzero torsion, the so-called $f(T)$ gravity where T signifies torsion. The acceleration of the universe in this gravity is caused by torsional forces. [25-27]. $f(T)$gravity is based on Weitzenbock geometry. Gravitation is attributed in this theory to the torsion of a space-time with zero curvature, which acts as a force [28].

Recently Chirde and Shekh [29] have investigated the development of dark energy parameter for spatially homogeneous and anisotropic Bianchi type-I universes within the context of $f(T)$ theory of gravity, employing a suitable physical assumption and Hubble's law of variation, which produces the constant value of the deceleration parameter. Daouda et al. [30] have created the $f(T)$ gravity model reconstruction using holographic dark energy. Dent et al. [31] have studied $f(T)$ cosmology at the levels of background and disturbance, and have developed a general framework in order to rebuild corresponding for any particular dynamical dark energy scenario, a one-parameter family of models. Jamil et al. [32] have investigated the model of dark energy interacting in $f(T)$

cosmology, considering dark energy to be a perfect fluid and selecting a specific cosmologically viable form $f(T) = \beta\sqrt{T}$. Sharif and Azeem [33] have explored the actions of the dark energy's state parameter and energy density equation in the setting of $f(T)$ gravity and used anisotropic LRS Bianchi type I universe model for this purpose. Sharif and Rani [34] have investigated the bulk viscosity of dust matter under generalized teleparallel gravity and analyzed many simulations of dark energy in this situation, as well as a viscous model with time dependence; the viscous equation of state parameter for these dark energy models must be developed.

We explored when dark energy is present in the Bianchi type-I cosmological model, utilising the theoretical framework, and were motivated by the earlier research. To acquire precise answers to the modified field equation of the gravitational teleparallel theory, we assumed a particular rule of variation for Hubble's parameter which produces a constant deceleration parameter. The following is how this document is structured, Section 2: $f(T)$gravity. Section 3: Equations using metric and field. Section 4: Solutions of field equations. Section 5: Some physical parameters. Section 6: Discussion. Section 7: Conclusion.

II. $f(T)$ GRAVITY

In the given section, we provides a quick explanation of $f(T)$ gravity as well as a full development of its field equations. The $f(T)$ theory of gravity is formulated in Weitzenböck space time using the line element made available by

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \tag{1}$$

where, $g_{\mu\nu}$ are the symmetric components having ten degrees of freedom.

It is possible to change this line element to Minkowski's description of the tetrad transformation as follows

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \eta_{ij}\theta^i\theta^j \tag{2}$$

$$dx^\mu = e_i^\mu\theta^i, \quad \theta^i = e_\mu^i dx^\mu \tag{3}$$

where, $e_i e_{j=}\eta_{ij}$ and $\eta_{ij} = diag(1,-1,-1,-1)$ is Minkowski metric.

The formula for the metric determinant's square root is $\sqrt{-g} = \det[e_\mu^i] = e$.

When just the non-zero torsion terms are present and the Riemann tensor component of the manifold is null (contribution of the Levi-Civita link), the components of the Weitzenböck connection are defined as,

$$\Gamma_{\mu\nu}^\rho = h_i^\rho\partial_\nu h_\mu^i = -h_\mu^i\partial_\nu h_i^\rho \tag{4}$$

The curvature is zero, but the torsion is not. The antisymmetric portion of this connection gives the following definitions of the tensor torsion's components,

$$T_{\mu\nu}^\rho = \Gamma^\rho{}_{\nu\mu} - \Gamma^\rho{}_{\mu\nu} = h_i^\rho(\partial_\mu h_\nu^i - \partial_\nu h_\mu^i) \tag{5}$$

The Levi-Civita connection and the Weitzenböck connection differ in a space-time tensor called the contortion tensor.

$$K_\rho^{\mu\nu} = -\frac{1}{2}(T^{\mu\nu}{}_\rho - T_\rho^{\nu\mu} - T_\rho{}^{\mu\nu}) \tag{6}$$

and antisymmetric tensor is,

$$S_\rho^{\mu\nu} = \frac{1}{2}(K^{\mu\nu}{}_\rho + \delta_\rho^\mu T^{\theta\nu}{}_\theta - \delta_\rho^\nu T^{\theta\mu}{}_\theta) \tag{7}$$

The form of the torsion scalar is

$$T = S_\rho^{\mu\nu}T_{\mu\nu}^\rho \tag{8}$$

A generalisation of the teleparallel theory of gravity, theory of gravity's field equation is given by the action, where T is the torsion scalar provided by [35,36]

$$I = \int e[f(T) = L_{matter}]d^4x \tag{9}$$

where, $f(T)$denotes Torsion scalar T's differentiable function and L_{matter} denotes the matter Lagrangian and $e = \sqrt{-g}$.

By altering the action with regard to the vierbein as follows, the teleparallel theory of gravity's modified field equation is discovered,

$$\left[e^{-1}\partial_\mu\left(eS_i{}^{\mu\nu}\right) - h_i^\lambda T^\rho{}_{\mu\lambda}S_\rho{}^{\nu\mu}\right]f_T + S_i^{\mu\nu}\partial_\mu(T)f_{TT} + \frac{1}{4}h_i^\nu f = \frac{1}{2}k^2 h_i^\rho\tau_\rho^\nu, \tag{10}$$

where, $S_i^{\mu\nu} = h_i^\rho S_\rho{}^{\mu\nu}, k^2 = 8\pi G, f_T = \frac{df}{dT}, \ f_{TT} = \frac{d^2f}{dT^2}$

III. EQUATIONS USING METRIC AND FIELD

The space time of Bianchi type-I is [37,38,39]

$$ds^2 = A^2\{dx^2 + dy^2 + (1 + \beta \int \frac{dt}{A^3})^2\, dz^2\} - dt^2 \quad (11)$$

where, β is a positive constant and A is proportional to cosmic time t.

Dark energy's energy momentum tensor is provided by

$$T_\rho^\nu = diag[\rho_m, -p_m, -p_m, -p_m] \quad (12)$$

where, p_m and ρ_m denote matter pressure and energy density, respectively.

For the dark energy of the energy momentum tensor in eqn. (12), the space time of Bianchi type-I may be expressed as follows from eqn. (10),

$$-2\left[\frac{\dot{A\beta}}{A^4(1+\beta\int\frac{dt}{A^3})}\right] f_T + \frac{(-\beta)}{A^3(1+\beta\int\frac{dt}{A^3})}\dot{T} f_{TT} - f = -k^2\rho_m \quad (13)$$

$$-2\left[\frac{\dot{A\beta}}{A^4(1+\beta\int\frac{dt}{A^3})}\right] f_T + \frac{(-\beta)}{A^3(1+\beta\int\frac{dt}{A^3})}\dot{T} f_{TT} - f = k^2 p_m \quad (14)$$

$$4\left[\frac{\ddot{A}}{A} + \frac{3\dot{A}^2}{A^2} + \frac{2\dot{A}\beta}{A^4(1+\beta\int\frac{dt}{A^3})^2} - \frac{\dot{A}^2}{A^2} - \frac{\dot{A\beta}}{A^4(1+\beta\int\frac{dt}{A^3})}\right] f_T - \frac{4\dot{A}}{A}\dot{T} f_{TT} - f = -2k^2 p_m \quad (15)$$

3. $$f - \frac{4\dot{A}^2}{A^2} - \frac{8\dot{A\beta}}{A^4(1+\beta\int\frac{dt}{A^3})} = -2k^2 p_m \quad (16)$$

Finally, there are four differential equations with five unknowns, specifically A, β, f, ρ_m, p_m.

From eqns. (6-8), we get Torsion Scalar T as,

$$T = \frac{-4\dot{A\beta}}{A^4(1+\beta\int\frac{dt}{A^3})} - \frac{2\dot{A}^2}{A^2} \quad (17)$$

and

$$\dot{T} = \frac{-4\ddot{A}A^4(1+\beta\int\frac{dt}{A^3}) + A\dot{\beta}^2}{[A^4(1+\beta\int\frac{dt}{A^3})]^2} - \left(\frac{4A^2\dot{A}\ddot{A} - 4A\dot{A}^2}{A^4}\right) \quad (18)$$

The Conservation equation is given as

$$p_m + 3H(\dot{\rho_m} + p_m) = 0 \quad (19)$$

IV. SOLUTIONS OF FIELD EQUATION

Assume that Hubble's parameter varies according to Bermann's special law of variation, which results in the constant deceleration parameter produced by the relation,

$$q = -\frac{a\ddot{a}}{\dot{a^2}} \quad (20)$$

here the average scale factor is represented by a.

The constant deceleration parameter has a negative sign since the universe is thought to be expanding faster.

From eqn. (1) of given metric,

$$a = [A^3(1 + \beta\int\frac{dt}{A^3})]^{\frac{1}{3}} \quad (21)$$

From eqns. (20) and (21), we get

$$a = (c_1 t + c_2)^{\frac{1}{1+q}};\ \ q \neq -1 \quad (22)$$

where $C_1 \neq 0$, C_2 are integrating constant.

The deceleration parameter q is given as,

$$q = n - 1 \quad (23)$$

Comparing eqns. (21) and (22)

$$A = (c_1 t + c_2)^{\frac{3}{n(m+3)}} \quad (24)$$

$$(1 + \beta\int\frac{dt}{A^3}) = (c_1 t + c_2)^{\frac{3m}{n(m+3)}} \quad (25)$$

Using eqns. (24) and (25) in the eqn. (11)

$$ds^2 = \{(c_1 t + c_2)^{\frac{3}{n(m+3)}}\}^2[dx^2 + dy^2 + \{(c_1 t + c_2)^{\frac{3m}{n(m+3)}}\}^2 dz^2] - dt^2 \quad (26)$$

From eqn. (16) pressure of the matter as follows,

$$p_m = -\frac{1}{2}k^2[f(T) - \frac{4\dot{A}^2}{A^2} - \frac{8\dot{A}\beta}{A^4(1+\beta\int\frac{dt}{A^3})}] \quad (27)$$

We take $f(T) = T$ in eqn. (27)

$$p_m = -\frac{1}{2}k^2[T - \frac{4\dot{A}^2}{A^2} - \frac{8\dot{A}\beta}{A^4(1+\beta\int\frac{dt}{A^3})}] \quad (28)$$

Where, $k^2 = 8\pi G$

The energy density-pressure relationship described by the EoS as,

$$p_m = \omega\rho_m \quad (29)$$

Where ω is EoS parameter

From eqns. (13) and (14)

$$\omega = -1 \quad (30)$$

From eqns. (29) and (30), we discover the following relationship between energy density and pressure,

$$p_m = -\rho_m \quad (31)$$

V. SOME PHYSICAL PARAMETERS

Dark energy is shown in eqn. (26), Universe described by a Bianchi Type I cosmological model within the context of the $f(T)$ theory of gravity. Now, using the physical parameters listed below, we will talk about the physical properties of model.

Mean Generalized Hubble's parameter is,

$$H = \frac{c_1C^{-1}}{b}(m+2) \quad (32)$$

Spatial Volume,

$$V = A^3(1+\beta\int\frac{dt}{A^3}) \quad (33)$$

Scalar expansion,

$$\theta = \frac{c_1C^{-1}}{b}(m+2) \quad (34)$$

The Anisotropy Parameter,

$$\Delta_m = \frac{2(m-1)^2}{(m+2)^2} \quad (35)$$

Shear Scalar,

$$\sigma^2 = \frac{3(m-1)^2c_1C^{-1}}{b(m+2)} \quad (36)$$

where, $b = n(m+3)$and $C = tc_1 + c_2$

VI. DISCUSSION

In this part, we use physical parameters to explain the model's physical behavior.

- We discovered that the Universe's spatial volume begins with the big bang at $t = -\frac{c_2}{c_1}$ and it always gets better as time goes on to expand and when t tends infinitely, the spatial volume tends indefinitely. This illustrates that the Universe has a starting volume of zero and grows over time.
- The mean generalized Hubble parameter, the scalar expansion and shear scalar are functions of cosmic time t. As t tends infinitely all these parameters are tends to zero and when $t = -\frac{c_2}{c_1}$ all these parameters are tends infinitely.
- Anisotropy parameter in independent of cosmic time and the anisotropy parameter have remained constant throughout the universe's evolution. The model does not reach isotropy when the anisotropy value is non-zero for $m \neq 1$, however the anisotropy parameter is zero for $m = 1$ and model is isotropic.

VII. CONCLUSION

Within this paper, we looked into the Bianchi type-I metric when there is dark energy present with the help of $f(T)$ theory of gravity. To solve the mathematical equations for the field, we suppose that the particular law of variation of Hubble's parameter produces a constant deceleration parameter. When $n < 1$ the deceleration parameter $q = -1 + n$results in an accelerating universe and when $n > 1$ the deceleration parameter $q = -1 + n$ results in a deceleration universe which matched the results of [3,40,41]. We found pressure and energy density relationship as, $p_m + \rho_m = 0$ with EoS parameter $\omega = -1$ gives an accelerating universe and $\omega = -1$ in our model epoch exits.

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