Realization of Electronically Tunable Current-Mode
Square-Root-Domain Multifunction Filter

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Abstract: In this study, a square-root domain, electronically adjustable current-mode, second-order multifunction filter circuit is designed. High-pass, band-reject and band-pass output currents can be obtained as the output current of the multifunction filter circuit. In addition, the cut-off frequency and Q quality factor of the filter circuit can be adjusted electronically by changing the values of the current I₀, dc in the circuit. PSPICE computer simulations were performed to verify the theoretical results obtained.

Keywords: Companding systems, current-mode circuits, square-root-domain filters

I. INTRODUCTION

It can be said that log domain and square-root domain filter circuits are the most common translinear circuits to which the companding technique is applied. The advantages of companding technology, such as wide dynamic range, low supply voltage and electronically adjustable design, increase the interest in this technique. The classical translinear principle or the bipolar translinear principle (BTL) [1, 2] uses the exponential current-voltage relationship of BJT’s or MOS transistors in the weak inversion region. The MOS translinear (MTL) principle developed by Seevinck using the BTL principle uses the quadrature current-voltage relationship of a MOS transistor operating in strong inversion or saturation region [3]. In the first phase of the square-root domain circuit design, the desired circuit function is transformed into state-space equations. At this stage, different sets of state-space equations representing the same circuit function can be generated. Each of these state-space equation sets corresponds to a different circuit having the same circuit function. In the second stage of the design, the current equations are obtained by using the quadratic current-voltage relation of the MOS transistors in the state-space equations. In the final stage, the resulting current equations are converted to square-root-domain circuits using analog processing blocks such as square-root and squarer/divider [5-10], current mirrors and dc current sources [4-20].

It can be seen that studies on first-order filter circuits [10-13], second-order filter circuits [7, 13-19] and third-order filter circuits [20] have been performed by various researchers when a literature survey on square-root domain circuits is carried out. There are also studies on trans-admittance and trans-impedance circuits [12,11,19]. In this study, a square-root domain second order current-mode multifunctional filter was implemented using state-space synthesis. High-pass, band-pass and band-reject output signals can be obtained from the output end of the implemented filter circuit. The f₀ cut-off frequency and the Q quality factor of the obtained output signals can be electronically adjusted by changing the current value of the dc current sources.

II. SRD MULTIFUNCTION FILTER

To implement the square-root domain multifunction filter design, let’s first consider the second order high-pass filter transfer function as given by general structure (1).

\[ F(s) = \frac{P(s)}{Q(s)} = \frac{s^2}{s^2 + \frac{a_0}{Q} s + \frac{a_b}{Q}} \]  

(1)

The high-pass transfer function given in (1) above can be transformed into state-space equations given in (2), (3) and (4) below.

\[ \dot{x}_1 = -\frac{a_b}{Q} x_1 - \frac{a_b}{Q} x_2 + \frac{a_b}{Q} u_1 \]  

(2)

\[ x_2 = \frac{a_b}{Q} x_1 + \frac{a_b}{Q} u_2 \]  

(3)

\[ y = u_2 - x_1 \]  

(4)
Here, the terms $x_1$ and $x_2$ represent the state variables, the term $y$ represents the output size of the filter circuit and $u_1$ and $u_2$ terms represent the input signals. $I_1$ and $I_2$, the drain currents of the MOS transistor in the saturation region which are represented by the state-variables $x_1$ and $x_2$ in the state-space equations can be defined as given in (5) and (6) \[11, 21\].

\[I_1 = \frac{\Theta}{2}(V_1 - V_{th})^2\]  \hspace{1cm} (5)

\[I_2 = \frac{\Theta}{2}(V_2 - V_{th})^2\]  \hspace{1cm} (6)

Here, the term $\Theta = \mu C_{ox}(W/L)$ is the conductivity value of MOS transistor, $V_1$ and $V_2$ voltages are the gate-to-source voltages and $V_{th}$ is the threshold voltage. In the currents (5) and (6) given above, (7) and (8) are obtained after taking derivatives of both sides.

\[I_1 = V_1 \sqrt{2\beta I_1}\]  \hspace{1cm} (7)

\[I_2 = V_2 \sqrt{2\beta I_2}\]  \hspace{1cm} (8)

In the next step, equations (7) and (8) are written in the state-space equations given in (2), (3) and (4) and both sides of the equations are multiplied by the $C$ constant. Then, necessary arrangements can be made to form (9) and (10) below.

\[CV_1 = -\frac{C_{ox}I_1}{\sqrt{2}} - \frac{C_{ox}I_2}{\sqrt{2}} + \frac{C_{ox}u_1}{\sqrt{2}}\]  \hspace{1cm} (9)

\[CV_2 = \frac{C_{ox}I_1}{\sqrt{2}} - \frac{C_{ox}I_2}{\sqrt{2}} + \frac{C_{ox}u_2}{\sqrt{2}}\]  \hspace{1cm} (10)

Here an $I_Q$ current can be defined as given in (11) \[7, 11\]. (9) and (10) are converted to current equations (12) and (13) below using this $I_Q$ current defined.

\[\sqrt{I_Q} = \frac{C_{ox}}{\sqrt{\beta}}\]  \hspace{1cm} (11)

\[CV_1 = -\frac{1}{Q} \frac{I_Q I_1}{2} + \frac{I_Q I_2}{2I_1} + \frac{1}{Q} \frac{I_Q u_1}{2I_1}\]  \hspace{1cm} (12)

\[CV_2 = \frac{I_Q I_1}{2I_2} - \frac{I_Q I_2}{2I_2}\]  \hspace{1cm} (13)

Finally, an $I_Q$ current as defined in (14) can be used in place of the $1/Q$ term in (12) and (13).

\[I_Q = \frac{1}{Q} I_0\]  \hspace{1cm} (14)

(14) can be used to set the value of quality factor $Q$. With the use of the defined $I_Q$ current, the final state of the current equations becomes as given in (15) and (16).

\[CV_1 = -\frac{I_Q I_1}{2} + \frac{I_Q I_2}{2I_1} + \frac{I_Q u_1}{2I_1}\]  \hspace{1cm} (15)

\[CV_2 = \frac{I_Q I_1}{2I_2} - \frac{I_Q I_2}{2I_2}\]  \hspace{1cm} (16)

Using the obtained current equations (15) and (16), the square-root domain second-order current-mode filter circuit can be implemented as shown in Fig.1.
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Figure 1. SRD multifunction filter.

Here, the input currents of the filter circuit are represented by \( I_{in1} \) and \( I_{in2} \) and the output currents are represented \( I_2 \) and \( I_3 \). Also, by means of (17), the cut-off frequency of the filter circuit \( \omega_0 \) can be determined depending on the values \( I_2 \), \( \beta \) and \( C \).

\[
\omega_0 = \frac{\sqrt{2}I_2}{C}
\]  

(17)

Since the input current source \( I_{in} \) has both dc and ac components, it can be formed by connecting the dc and ac current sources in parallel, as shown Fig. 2 [10].

Figure 2. Input current sources.

Using the state-space equations (2) and (3), the output of the multifunction filter circuit the currents \( I_1 \) and \( I_2 \) can be obtained as given in (18) and (19) below depending on \( \omega_0 \) and \( Q \). Here, \( I_{in2} = I_{in1} = I_{in} \) is taken so that high-pass output can be obtained.

\[
I_1 = \frac{s^2 + \omega_0^2}{s^2 + \frac{2}{Q} + \omega_0^2} I_{in}
\]

(18)

\[
I_2 = \frac{\omega_0^2}{s^2 + \frac{2}{Q} + \omega_0^2} I_{in}
\]

(19)

Here, the \( I_4 \) current given in (18) is obtained as given by the high-pass filter current output (20) by being used in the definition of the multifunction filter output current given by (4).

\[
I_{HP} = I_{in} - I_1 = \frac{s^2 + \omega_0^2}{s^2 + \frac{2}{Q} + \omega_0^2} I_{in}
\]

(20)

To obtain the second-order current-mode band reject filter output current, we take \( u_2 = 0 \) in the state-space equation (2) and (3). The new equations obtained, in this case, will be as given in (21) and (22).

\[
\dot{x}_1 = -\frac{\omega_0}{Q} x_1 - \omega_0 x_2 + \frac{\omega_0}{Q} u_1
\]

(21)
\[
\dot{x}_2 = ax_1 \quad (22)
\]

Here, current equations (23) and (24) can be obtained by using the MOS transistor saturation region currents \(I_1\) and \(I_2\) instead of the \(x_1\) and \(x_2\) state variables; and by making the necessary adjustments.

\[
C_{ij} I_1 = \frac{I_{Q1}}{2} + \frac{I_{Q2}}{2I_1} + \frac{I_{Q3}}{2I_1} \quad (23)
\]

\[
C_{ij} I_2 = \frac{I_{Q2}}{2I_2} \quad (24)
\]

Equations (24) and (24) given above correspond to the case of \(I_{\text{in}2(\text{ac})} = 0\) in the circuit given in Fig.1. For this case \(I_1\) and \(I_2\) currents are obtained as given in (25) and (26) depending on \(a_0\) and \(Q\). It can be said here that \(I_{\text{in1}} = I_{\text{in}}\) and \(I_{\text{in2}} = 0\) are taken in order to obtain the band-reject output.

\[
I_1 = \frac{\frac{a_0}{Q}}{s^2 + \frac{a_0}{Q} s + \frac{a_0^2}{Q}} I_{\text{in}} \quad (25)
\]

\[
I_2 = \frac{\frac{a_0}{Q}}{s^2 + \frac{a_0}{Q} s + \frac{a_0^2}{Q}} I_{\text{in}} \quad (26)
\]

In this case, the current \(I_{BR}\) given in (26) can be obtained as given in the band-reject filter output current (27), after it is used in the definition of the multifunction filter output current given in (4).

\[
I_{BR} = I_{\text{in}} - I_1 = \frac{s^2 - \frac{a_0}{Q}}{s^2 + \frac{a_0}{Q} s + \frac{a_0^2}{Q}} I_{\text{in}} \quad (27)
\]

Furthermore, the current \(I_{BP}\) itself corresponds to a band-pass output current as given in (28).

\[
I_{BP} = I_1 = \frac{\frac{a_0}{Q}}{s^2 + \frac{a_0}{Q} s + \frac{a_0^2}{Q}} I_{\text{in}} \quad (28)
\]

Thus, as the output current of the same circuit given in Fig.1, it is possible to obtain high-pass, band-reject and band-pass output currents as defined in Table.1.

### Table.1 Different output currents of multi-function filter

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Type of filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I_{\text{in}1(\text{ac})})</td>
<td>(I_{\text{in}2(\text{ac})})</td>
<td>(y = u - x_1)</td>
</tr>
<tr>
<td>(I_{\text{in}})</td>
<td>(I_{\text{in}})</td>
<td>(I_{\text{out}} = I_{\text{in}} - I_1)</td>
</tr>
<tr>
<td>(I_{\text{in}})</td>
<td>0</td>
<td>(I_{\text{out}} = I_{\text{in}} - I_1)</td>
</tr>
<tr>
<td>(I_{\text{in}})</td>
<td>0</td>
<td>(I_{\text{out}} = I_1)</td>
</tr>
</tbody>
</table>

### III. SIMULATION RESULTS

The PSPICE simulation of the proposed square-root domain second order current-mode multi-function filter circuit were performed using TSMC 0.35\(\mu\)m Level3 CMOS transistor parameters [22]. The sizes of the transistor used in the circuit were chosen as \(W/L = 10\mu\text{m}/10\mu\text{m}\) for \(M_1\sim M_7\) transistors and \(W/L = 220\mu\text{m}/2\mu\text{m}\) for \(M_8\sim M_{15}\) transistors. \(V_{\text{DD}} = 3\text{V}\) and \(C = 300\text{pF}\) were taken for the supply voltage and capacitors values used in the circuit. Gain responses were obtained for different cut-off frequency values of the multifunctional filter circuit. For this purpose, it was observed that when the value of \(I_{\text{d}}\) current sources is changed between \(2.8\mu\text{A}\sim 140.2\mu\text{A}\), the cut-off frequency of the filter circuit changes.
between 5.5kHz~45kHz. The gain response curves for the different cut-off frequency values of the multifunction filter are shown in Fig.3.

![Figure 3: Gain responses of high-pass filter at different values of $I_0$.](image-url)

The gain response curves for the band-reject and band-pass outputs of the different cut-off frequency values of the multifunctional filter circuit are shown in Fig.4 and Fig.5, respectively.

![Figure 4: Gain responses of band-reject filter at different values of $I_0$.](image-url)

![Figure 5: Gain responses of band-pass filter at different values of $I_0$.](image-url)
When an input signal with a frequency of \(25kHZ\) and a peak amplitude of \(20\mu A\) is applied to the input of the multifunction filter circuit, the time-dependent changes of the input current and the output current obtained from the band-pass output for \(I_0 = 23.2\mu A\) are obtained as shown in Fig.6.

![Figure 6. Time domain responses of band-pass filter.](image)

When the amplitude peak value of the signal with frequency value of \(25kHZ\) applied to the input of the multifunction filter circuit is changed within the range of \(15\mu A \sim 100\mu A\), the total harmonic distortion (THD) occurring in the band-pass output current is less than \(2\%\). The power consumption of the multifunctional filter circuit is on the level of \(25mW\).

IV. CONCLUSION

In this study, a square-root domain second order current-mode multifunction filter circuit was implemented using a state-space synthesis method. High-pass, band-reject and band-pass current outputs can be obtained from this multifunction filter circuit. In the filter circuit, square-root and squarer/divider analogue processing blocks with square-root domains as well as current mirrors and dc current sources were used. Apart from these, two grounded capacitors and one dc voltage source were used. The cut-off frequency \(f_0\) and the quality factor of the filter circuit can be adjusted electronically by changing the values of the \(I_0\) dc current sources. By changing the current values of the dc current sources \(I_0\), gain responses with different cut-off frequency values were obtained. The theoretical results obtained were verified by PSPICE simulations.

REFERENCES

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