Production Inventory Model with Different Deterioration Rates Under Shortages and Linear Demand

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Abstract:- A production inventory model with linear trend in demand is developed. Different deterioration rates are considered in a cycle. Holding cost is considered as function of time. Shortages are allowed. To illustrate the model numerical example is provided and sensitivity analysis is also carried out for parameters.

Keywords:- Production, Inventory model, Varying Deterioration, Linear demand, Time varying holding cost, Shortages

I. INTRODUCTION

Deterioration of an item is defined as damage, spoilage, dryness, vaporization, etc. that decreases the utility of the item. Ghare and Schrader [1963] first developed an EOQ model with constant rate of deterioration. The model was extended by Covert and Philip [1973] by considering variable rate of deterioration. The model was further extended by Shah and Jaiswal [1977] by considering shortages. Gupta and Vrat [1986] developed an inventory model with stock dependent consumption rate. Giri et al. [1996] developed inventory model with stock dependent demand. The related works are found in (Raafat [1991], Goyal and Giri [2001], Ruxian et al. [2010]).

Mandal and Phaujdar [1989] developed production inventory model for deteriorating items with linear stock dependent demand. Roy and Chaudhary [2009] studied a production inventory model for deteriorating items when demand rate is inventory level dependent and production rate depends both on stock level and demand. Tripathy and Mishra [2011] developed an order level inventory model with time dependent Weibull deterioration and ramp type demand rate where production and demand were time dependent. Patel and Patel [2013] developed a deteriorating items production inventory model with demand dependent production rate. Sharma and Chaudhary [2013] dealt with inventory model for deteriorating items with time dependent demand rate. Shortages were allowed and completely backlogged. Patel and Sheikh [2015] developed an inventory model with different deterioration rates under linear demand and time varying holding cost.

In most of the products, initially there is no deterioration. After certain time deterioration starts and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory models.

A production inventory model with different deterioration rates for the cycle time under time varying holding cost is developed in this paper. Shortages are allowed and completely backlogged. Numerical example is provided to illustrate the model and sensitivity analysis of the optimal solutions for major parameters is also carried out.

II. ASSUMPTIONS AND NOTATIONS

NOTATIONS:

The following notations are used for the development of the model:

- P(t): Production rate is a function of demand ($P(t) = \eta D(t), \eta > 0$)
- D(t) : Demand rate is a linear function of time t (a+bt, a>0, 0<b<1)
- A : Replenishment cost per order
- c : Purchasing cost per unit
- p : Selling price per unit
- T : Length of inventory cycle
- I(t) : Inventory level at any instant of time t, $0 \le t \le T$
- Q_1 : Order quantity initially
- Q_2 : Quantity of shortages
- Q : Order quantity
- c_2 : Shortage cost per unit
- θ : Deterioration rate during $\mu_1 \le t \le t_1, 0 < \theta < 1$

- θ t : Deterioration rate during , $t_1 \le t \le t_0$, $0 < \theta < 1$
- π : Total relevant profit per unit time.

ASSUMPTIONS:

The following assumptions are considered for the development of two warehouse model.

- The demand of the product is declining as a linear function of time.
- Rate of production is a function of demand
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are allowed and completely backlogged.
- Deteriorated units neither be repaired nor replaced during the cycle time.

III. THE MATHEMATICAL MODEL AND ANALYSIS

Let I(t) be the inventory at time t ($0 \le t \le T$) as shown in figure.

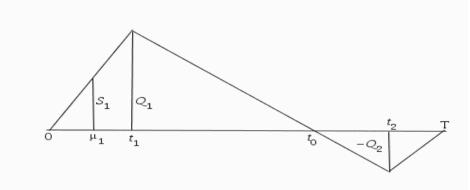


Figure 1

The differential equations which describes the instantaneous states of I(t) over the period (0, T) is given by

$$\frac{\mathrm{dI}(t)}{\mathrm{dt}} = (\eta - 1)(a + bt), \qquad \qquad 0 \le t \le \mu_1 \quad (1)$$

$$\frac{dI(t)}{dt} + \theta I(t) = (\eta - 1)(a + bt), \qquad \mu_1 \le t \le t_1 \quad (2)$$

$$\frac{dI(t)}{dt} + \theta tI(t) = -(a + bt), \qquad t_1 \le t \le t_0 \quad (3)$$

$$\frac{\mathrm{dI}(t)}{\mathrm{dt}} = -(a+bt), \qquad \qquad t_0 \le t \le t_2 \quad (4)$$

$$\frac{\mathrm{dI}(t)}{\mathrm{dt}} = (\eta - 1)(a + bt), \qquad t_2 \le t \le T \qquad (5)$$

with initial conditions I(0) = 0, $I(\mu_1) = S_1$, $I(t_1) = Q_1$, $I(t_0) = 0$, $I(t_2) = -Q_2$, and I(T) = 0. Solutions of these equations are given by

$$I(t) = (\eta - 1)(at + \frac{1}{2}bt^{2}).$$
(6)

$$I(t) = S_1 \left[1 + \theta(\mu_1 - t) \right]$$

$$(n + 1) \left[2(\mu_1 - t) + \frac{1}{2} b(\mu_2^2 - t^2) + \frac{1}{2} 2\theta(\mu_2^2 - t^2) + \frac{1}{2} b\theta(\mu_3^3 - t^3) + 2\theta(\mu_1 - t) + \frac{1}{2} b\theta(\mu_2^2 - t^2) \right]$$
(7)

$$-(\eta-1)\left[a(\mu_{1}-t)+\frac{1}{2}b(\mu_{1}^{2}-t^{2})+\frac{1}{2}a\theta(\mu_{1}^{2}-t^{2})+\frac{1}{3}b\theta(\mu_{1}^{3}-t^{3})-a\theta t(\mu_{1}-t)-\frac{1}{2}b\theta t(\mu_{1}^{2}-t^{2})\right]$$

$$\mathbf{I}(t) = \left[a(t_0 - t) + \frac{1}{2}b(t_0^2 - t^2) + \frac{1}{6}a\theta(t_0^3 - t^3) + \frac{1}{8}b\theta(t_0^4 - t^4) - \frac{1}{2}a\theta t^2(t_0 - t) - \frac{1}{4}b\theta t^2(t_0^2 - t^2)\right]$$
(8)

$$I(t) = \left[a(t_0 - t) + \frac{1}{2}b(t_0^2 - t^2) \right]$$
(9)

$$I(t) = (\eta - 1) \left[a(t - T) + \frac{1}{2}b(t^2 - T^2) \right]$$
(10)

(by neglecting higher powers of θ)

Putting $t = \mu_1$ in equation (6), we get

$$S_{1} = (\eta - 1)(a\mu_{1} + \frac{1}{2}b\mu_{1}^{2})$$
(11)

Putting $t = t_1$ in equations (7) and (8), we have $I(t_1) = S \cdot [1 + \theta(\mu_1 - t_1)]$

$$f_{1} = S_{1} \left[1 + \theta(\mu_{1} - t_{1}) \right] - (\eta - 1) \left[a(\mu_{1} - t_{1}) + \frac{1}{2} b(\mu_{1}^{2} - t_{1}^{2}) + \frac{1}{2} a\theta(\mu_{1}^{2} - t_{1}^{2}) + \frac{1}{3} b\theta(\mu_{1}^{3} - t_{1}^{3}) - a\theta t_{1}(\mu_{1} - t_{1}) - \frac{1}{2} b\theta t_{1}(\mu_{1}^{2} - t_{1}^{2}) \right]$$
(12)

$$\mathbf{I}(\mathbf{t}_{1}) = \left[a\left(\mathbf{t}_{0} - \mathbf{t}_{1}\right) + \frac{1}{2}b\left(\mathbf{t}_{0}^{2} - \mathbf{t}_{1}^{2}\right) + \frac{1}{6}a\theta\left(\mathbf{t}_{0}^{3} - \mathbf{t}_{1}^{3}\right) + \frac{1}{8}b\theta\left(\mathbf{t}_{0}^{4} - \mathbf{t}_{1}^{4}\right) - \frac{1}{2}a\theta\mathbf{t}_{1}^{2}\left(\mathbf{t}_{0} - \mathbf{t}_{1}\right) - \frac{1}{4}b\theta\mathbf{t}_{1}^{2}\left(\mathbf{t}_{0}^{2} - \mathbf{t}_{1}^{2}\right)\right].$$
(13)

So, from equations (8) and (9), we have

$$t_{1} = \frac{S_{1}(1 + \theta\mu_{1}) - (\eta - 1)a\mu_{1} - at_{0}}{S_{1}\theta - \eta a}$$
(14)

From equation (14), we see that t_1 is a function of μ_1 , T and S_1 , so t_1 is not a decision variable. Similarly putting $t = t_2$ in equations (9) and (10), we have

$$\mathbf{I}(\mathbf{t}_{2}) = \left[\mathbf{a} \left(\mathbf{t}_{0} - \mathbf{t}_{2} \right) + \frac{1}{2} \mathbf{b} \left(\mathbf{t}_{0}^{2} - \mathbf{t}_{2}^{2} \right) \right]$$
(15)

$$I(t_{2}) = (\eta - 1) \left[a(t_{2} - T) + \frac{1}{2} b(t_{2}^{2} - T^{2}) \right]$$
(16)

So, from equations (15) and (16), we have

$$t_{2} = \frac{t_{0} + (\eta - 1)T}{\eta}$$
(17)

From equation (17), we see that t_2 is a function of t_0 and T, so t_2 is not a decision variable. Putting $t = t_1$ in equation (7), we get

$$Q_{1} = (\eta - 1)(a\mu_{1} + \frac{1}{2}b\mu_{1}^{2})\left[1 + \theta(\mu_{1} - t_{1})\right] - (\eta - 1)\left[a(\mu_{1} - t_{1}) + \frac{1}{2}b(\mu_{1}^{2} - t_{1}^{2}) + \frac{1}{2}a\theta(\mu_{1}^{2} - t_{1}^{2}) + \frac{1}{3}b\theta(\mu_{1}^{3} - t_{1}^{3}) - a\theta t_{1}(\mu_{1} - t_{1}) - \frac{1}{2}b\theta t_{1}(\mu_{1}^{2} - t_{1}^{2})\right]$$
(18)

Similarly, putting $t = t_2$ in equation (9), we get

$$Q_{2} = \left[a(t_{2} - t_{0}) + \frac{1}{2}b(t_{2}^{2} - t_{0}^{2})\right]$$
(19)

Putting value of S_1 from equation (11) in equation (7), we have

$$I(t) = (\eta - 1)(a\mu_1 + \frac{1}{2}b\mu_1^2)\left[1 + \theta(\mu_1 - t)\right]$$

$$- (\eta - 1)\left[a(\mu_1 - t) + \frac{1}{2}b(\mu_1^2 - t^2) + \frac{1}{2}a\theta(\mu_1^2 - t^2) + \frac{1}{3}b\theta(\mu_1^3 - t^3) - a\theta t(\mu_1 - t) - \frac{1}{2}b\theta t(\mu_1^2 - t^2)\right]$$
(20)

Based on the assumptions and descriptions of the model, the total annual relevant profit (π), include the following elements: (i) Ordering cost (OC) = A

(1) Ordering cost (OC) = A
(21)
(ii) HC =
$$\int_{0}^{t_0} (x+yt)I(t)dt$$

= $\int_{0}^{\mu_1} (x+yt)I(t)dt + \int_{\mu_1}^{t_1} (x+yt)I(t)dt + \int_{t_1}^{t_0} (x+yt)I(t)dt$

$$\begin{split} &= -x \bigg(at_0 + \frac{1}{2} bt_0^2 + \frac{1}{6} a\theta t_0^3 + \frac{1}{8} b\theta t_0^4 \bigg) t_1 + \frac{1}{4} y \bigg(\frac{1}{2} \eta b - \frac{1}{2} b \bigg) \mu_1^4 + \frac{1}{2} x \big(\eta a - a \big) \mu_1^2 - \frac{1}{3} \bigg(x \bigg(-\frac{1}{2} b - \frac{1}{2} a\theta t_0 - \frac{1}{4} b\theta t_0^2 \bigg) - ya \bigg) t_1^3 \\ &+ \frac{1}{3} \bigg(x \bigg(-\frac{1}{2} b - \frac{1}{2} a\theta t_0 - \frac{1}{4} b\theta t_0^2 \bigg) - ya \bigg) t_0^3 - x \bigg(\frac{1}{2} \eta a\theta \mu_1^2 + \frac{1}{6} \eta b\theta \mu_1^3 - \frac{1}{2} a\theta \mu_1^2 - \frac{1}{6} b\theta \mu_1^3 \bigg) \mu_1 + \frac{1}{5} \bigg(\frac{1}{8} xb\theta + \frac{1}{3} ya\theta \bigg) t_0^5 \bigg) \\ &+ \frac{1}{4} \bigg(\frac{1}{3} xa\theta + y \bigg(-\frac{1}{2} b - \frac{1}{2} a\theta t_0 - \frac{1}{4} b\theta t_0^2 \bigg) \bigg) t_0^4 + \frac{1}{2} \bigg(-xa + y \bigg(at_0 + \frac{1}{2} bt_0^2 + \frac{1}{6} a\theta t_0^3 + \frac{1}{8} b\theta t_0^4 \bigg) \bigg) t_0^4 - \frac{1}{5} \bigg(\frac{1}{8} xb\theta + \frac{1}{3} ya\theta \bigg) t_1^5 \\ &+ \frac{1}{4} \bigg(x \bigg(-\frac{1}{6} \eta b\theta + \frac{1}{6} b\theta \bigg) + y \bigg(-\frac{1}{2} b + \frac{1}{2} \eta b + \frac{1}{2} a\theta - \frac{1}{2} \eta a\theta \bigg) \bigg) t_1^4 + \frac{1}{2} \bigg(x \big(\eta a - a \big) + y \bigg(\frac{1}{2} \eta a\theta \mu_1^2 + \frac{1}{6} \eta b\theta \mu_1^3 - \frac{1}{2} a\theta \mu_1^2 - \frac{1}{6} b\theta \mu_1^3 \bigg) \bigg) t_1^2 \\ &+ \frac{1}{3} \bigg(x \bigg(\frac{1}{2} \eta b - \frac{1}{2} b \bigg) + y \big(\eta a - a \big) \bigg) \mu_1^3 \end{split}$$

$$\begin{aligned} &-\frac{1}{48} yb\theta t_{1}^{6} - \frac{1}{4} \left(x \left(-\frac{1}{6} \eta b\theta + \frac{1}{6} b\theta \right) + y \left(-\frac{1}{2} b + \frac{1}{2} \eta b + \frac{1}{2} a\theta - \frac{1}{2} \eta a\theta \right) \right) \mu_{1}^{4} - \frac{1}{2} \left(-xa + y \left(-\frac{1}{2} b - \frac{1}{2} a\theta t_{0} - \frac{1}{4} b\theta t_{0}^{2} \right) \right) t_{1}^{4} \\ &-\frac{1}{2} \left(x \left(\eta a - a \right) + y \left(\frac{1}{2} \eta a\theta \mu_{1}^{2} + \frac{1}{6} \eta b\theta \mu_{1}^{3} - \frac{1}{2} a\theta \mu_{1}^{2} - \frac{1}{6} b\theta \mu_{1}^{3} \right) \right) \mu_{1}^{2} + x \left(\frac{1}{2} \eta a\theta \mu_{1}^{2} + \frac{1}{6} \eta b\theta \mu_{1}^{3} - \frac{1}{2} a\theta \mu_{1}^{2} - \frac{1}{6} b\theta \mu_{1}^{3} \right) t_{1} \\ &-\frac{1}{2} \left(x \left(-\frac{1}{2} b + \frac{1}{2} \eta b + \frac{1}{2} a\theta - \frac{1}{2} \eta a\theta \right) + y \left(\eta a - a \right) \right) \mu_{1}^{3} + \frac{1}{5} y \left(-\frac{1}{6} \eta b\theta + \frac{1}{6} b\theta \right) t_{1}^{5} - \frac{1}{5} y \left(-\frac{1}{6} \eta b\theta + \frac{1}{6} b\theta \right) \mu_{1}^{5} \\ &+ \frac{1}{3} \left(x \left(-\frac{1}{2} b + \frac{1}{2} \eta b + \frac{1}{2} a\theta - \frac{1}{2} \eta a\theta \right) + y \left(\eta a - a \right) \right) t_{1}^{3} + x \left(at_{0} + \frac{1}{2} bt_{0}^{2} + \frac{1}{6} a\theta t_{0}^{3} + \frac{1}{8} b\theta t_{0}^{4} \right) t_{0} + \frac{1}{48} yb\theta t_{0}^{6} \\ & (by neglecting higher powers of \theta) \end{aligned}$$

$$\begin{array}{ll} (iii) \quad \mathbf{DC} = c \left(\int\limits_{\mu_{1}}^{t_{1}} \theta I(t) dt + \int\limits_{\tau_{1}}^{\tau_{0}} \theta I(t) dt \right) \\ = c \left(\begin{array}{l} \eta a \mu_{1} t_{1} + \frac{1}{2} \eta b \mu_{1}^{2} t_{1} - a \mu_{1} t_{1} - \frac{1}{2} b \mu_{1}^{2} t_{1} + \eta a \theta \mu_{1} \left(\mu_{1} t_{1} - \frac{1}{2} t_{1}^{2} \right) + \frac{1}{2} \eta b \theta \mu_{1}^{2} \left(\mu_{1} t_{1} - \frac{1}{2} t_{1}^{2} \right) - a \theta \left(\frac{1}{2} \mu_{1} t_{1}^{2} - \frac{1}{3} t_{1}^{3} \right) \\ - a \theta \mu_{1} \left(\mu_{1} t_{1} - \frac{1}{2} t_{1}^{2} \right) - \frac{1}{2} b \theta \mu_{1}^{2} \left(\mu_{1} t_{1} - \frac{1}{2} t_{1}^{2} \right) - \eta a \left(\mu_{1} t_{1} - \frac{1}{2} t_{1}^{2} \right) - \frac{1}{2} \eta b \left(\mu_{1}^{2} t_{1} - \frac{1}{3} t_{1}^{3} \right) - \frac{1}{2} b \theta \left(\frac{1}{2} \mu_{1}^{2} t_{1}^{2} - \frac{1}{4} t_{1}^{4} \right) \\ - \frac{1}{2} \eta a \theta \left(\mu_{1}^{2} t_{1} - \frac{1}{3} t_{1}^{3} \right) - \frac{1}{3} \eta b \theta \left(\mu_{1}^{3} t_{1} - \frac{1}{4} t_{1}^{4} \right) + \eta a \theta \left(\frac{1}{2} \mu_{1} t_{1}^{2} - \frac{1}{3} t_{1}^{3} \right) + \frac{1}{2} \eta b \theta \left(\frac{1}{2} \mu_{1}^{2} t_{1}^{2} - \frac{1}{4} t_{1}^{4} \right) \\ + a \left(\mu_{1} t_{1} - \frac{1}{2} t_{1}^{2} \right) + \frac{1}{2} b \left(\mu_{1}^{2} t_{1} - \frac{1}{3} t_{1}^{3} \right) + \frac{1}{2} a \theta \left(\mu_{1}^{2} t_{1} - \frac{1}{3} t_{1}^{3} \right) + \frac{1}{2} \eta b \theta \left(\mu_{1}^{3} t_{1} - \frac{1}{4} t_{1}^{4} \right) \\ - c \theta \left(\frac{1}{2} \eta a \mu_{1}^{2} + \frac{1}{6} \eta b \mu_{1}^{3} - \frac{1}{2} a \mu_{1}^{2} - \frac{1}{6} b \mu_{1}^{3} + \frac{1}{3} \eta a \theta \mu_{1}^{3} + \frac{1}{8} \eta b \theta \mu_{1}^{4} - \frac{1}{3} a \theta \mu_{1}^{3} + \frac{1}{8} b \theta \left(\mu_{1}^{3} t_{1} - \frac{1}{4} t_{1}^{4} \right) \\ - c \theta \left(\frac{1}{2} \eta a \mu_{1}^{2} + \frac{1}{6} \eta b \mu_{1}^{3} - \frac{1}{2} a \mu_{1}^{2} - \frac{1}{6} b \mu_{1}^{3} + \frac{1}{3} \eta a \theta \mu_{1}^{3} + \frac{1}{8} \eta b \theta \mu_{1}^{4} - \frac{1}{3} a \theta \mu_{1}^{3} + \frac{1}{8} b \theta h_{1}^{4} \right) \\ + c \theta \left(\frac{1}{48} b \theta t_{0}^{6} + \frac{1}{15} a \theta t_{0}^{5} + \frac{1}{4} \left(-\frac{1}{2} b - \frac{1}{2} a \theta t_{0} - \frac{1}{4} b \theta t_{0}^{2} \right) t_{0}^{4} - \frac{1}{3} a t_{1}^{3} + \frac{1}{2} \left(a t_{0} + \frac{1}{2} b t_{0}^{2} + \frac{1}{6} a \theta t_{0}^{3} + \frac{1}{8} b \theta t_{0}^{4} \right) t_{0}^{2} \right) \end{array} \right) \\ - c \theta \left(\frac{1}{48} b \theta t_{0}^{6} + \frac{1}{15} a \theta t_{1}^{5} + \frac{1}{4} \left(-\frac{1}{2} b - \frac{1}{2} a \theta t_{0} - \frac{1}{4} b \theta t_{0}^{2} \right) t_{1}^{4} - \frac{1}{3} a t_{1}^{4} + \frac{1}{2} \left(a t_{0} + \frac{1}{2} b t_{0}^{2} + \frac{1}{6} a \theta t_{0}^{$$

(iv) Shortage cost is given by (T)

$$SC = -c_{2} \left(\int_{t_{0}}^{T} I(t) dt \right) = -c_{2} \left(\int_{t_{0}}^{t_{2}} I(t) dt + \int_{t_{2}}^{T} I(t) dt \right)$$

$$= -c_{2} \left(-\frac{1}{6} b \left(\frac{(t_{0} + \eta T - T)^{3}}{\eta^{3}} - t_{0}^{3} \right) - \frac{1}{2} a \left(\frac{(t_{0} + \eta T - T)^{2}}{\eta^{2}} - t_{0}^{2} \right) + at_{0} \left(\frac{(t_{0} + \eta T - T)}{\eta} - t_{0} \right) + \frac{1}{2} b t_{0}^{2} \left(\frac{(t_{0} + \eta T - T)}{\eta} - t_{0} \right) \right)$$

$$- c_{2} \left(\frac{1}{3} \left(\frac{1}{2} \eta b - \frac{1}{2} b \right) \left(T^{3} - \frac{(t_{0} + \eta T - T)^{3}}{\eta^{3}} \right) + \frac{1}{2} (\eta a - a) \left(T^{2} - \frac{(t_{0} + \eta T - T)^{2}}{\eta^{2}} \right) - \eta a T \left(T - \frac{(t_{0} + \eta T - T)}{\eta} \right) \right)$$

$$(24)$$

$$- c_{2} \left(\frac{1}{2} \eta b T^{2} \left(T - \frac{(t_{0} + \eta T - T)}{\eta} \right) + a T \left(T - \frac{(t_{0} + \eta T - T)}{\eta} \right) + \frac{1}{2} b T^{2} \left(T - \frac{(t_{0} + \eta T - T)}{\eta} \right) \right)$$

(v)
$$SR = p\left(\int_{0}^{T} (a+bt)dt\right) = p\left(aT + \frac{1}{2}bT^{2}\right)$$
 (25)

The total profit during a cycle, $\pi(t_0, T)$ consisted of the following:

$$\pi(t_0, T) = \frac{1}{T} \left[SR - OC - HC - DC - SC \right]$$
(26)

Substituting values from equations (21) to (25) in equation (26), we get total profit per unit.

The optimal value of $t_0 = t_0^*$ and $T = T^*$ (say), which maximizes profit $\pi(t_0,T)$ can be obtained by differentiating it with respect to t_0 and T and equate it to zero

i.e.
$$\frac{\partial \pi(t_0, T)}{\partial t_0} = 0, \ \frac{\partial \pi(t_0, T)}{\partial T} = 0$$
 (27)

provided it satisfies the condition

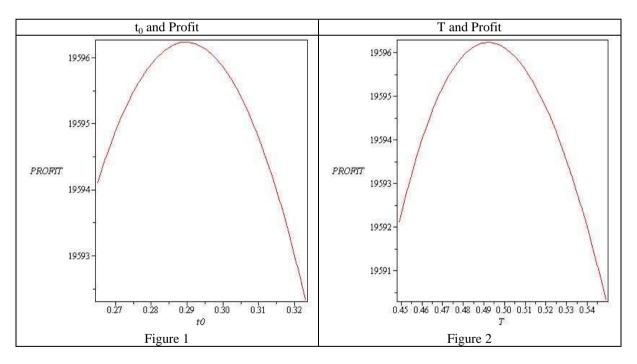
$$\frac{\partial^2 \pi(\mathbf{t}_0, \mathbf{T})}{\partial t_0^2} = \frac{\partial^2 \pi(\mathbf{t}_0, \mathbf{T})}{\partial t_0 \partial \mathbf{T}} > 0.$$

$$\frac{\partial^2 \pi(\mathbf{t}_0, \mathbf{T})}{\partial \mathbf{T} \partial t_0} = \frac{\mathbf{d}^2 \pi(\mathbf{t}_0, \mathbf{T})}{\partial \mathbf{T}^2} > 0.$$
(28)

IV. NUMERICAL EXAMPLE

Considering A= Rs.100, a = 500, b=0.05, η =2, c=Rs. 25, p= Rs. 40, θ =0.05, x = Rs. 5, y=0.05, c₂ = Rs. 8, μ_1 =0.30 t₀, in appropriate units. The optimal value of t₀* = 0.2897, T* = 0.4920 and Profit*= Rs. 19596.2392. The second order conditions given in equation (28) are also satisfied. The graphical representation of

the concavity of the profit function is also given.



V. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1 Sensitivity Analysis					
Parameter	%	t ₀	Т	Profit	
a	+20%	0.2650	0.4496	23557.8724	
	+10%	0.2765	0.4694	21576.6181	
	-10%	0.3049	0.5183	17616.8711	
	-20%	0.3229	0.5494	15638.6883	
θ	+20%	0.2855	0.4889	19594.0682	
	+10%	0.2875	0.4905	19595.1463	
	-10%	0.2918	0.4936	19597.3473	
	-20%	0.2940	0.4952	19598.4712	
x	+20%	0.2583	0.4703	19576.8036	
	+10%	0.2730	0.4805	19586.0742	
	-10%	0.3086	0.5063	19607.4409	
	-20%	0.3303	0.5209	19619.8552	
А	+20%	0.3165	0.5383	19557.4192	
	+10%	0.3034	0.5157	19576.3934	
	-10%	0.2752	0.4671	19617.0910	
	-20%	0.2598	0.4470	19639.1220	

Table	1	Sensitivity	Analysis
Lanc		Scholing	Analysis

From the table we observe that as parameter a increases/ decreases average total profit and optimum order quantity also increases/ decreases.

Also, we observe that with increase and decrease in the value of θ and x, there is corresponding decrease/increase in total profit and optimum order quantity.

From the table we observe that as parameter A increases/ decreases average total profit decreases/ increases and optimum order quantity increases/ decreases.

VI. CONCLUSION

In this paper, we have developed a production inventory model for deteriorating items with linear demand with different deterioration rates and shortages. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

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