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Computational Study of Degeneracy in Initial Basic Feasible Solution for the Transportation Problem

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Abstract:- The Transportation Problem is a classic problem of mathematical programming and may be applied to a variety of real life problems. Not only is it used in commodity transportation problems, but also in other applications. The objective in the problem is to determine the amounts shipped from each source to each destination minimizing the total ship cost, respecting supply and demand constraints. The algorithms to solve the problem usually start with an initial basic feasible solution and then solution is iteratively improved. In order to obtain the initial solution the three most cited methods in the literature are Northwest Corner Rule, the Least Cost Method and the Vogel Method. The problem is that the initial feasible solution may not be basic and then the basis needs to be completed with degenerated variables. This paper presents a study of degeneracy occurrence when the Least Cost Method is used to obtain the initial feasible solution, comparing different problem sizes, and ranges of cost, supplies and demands.

Keywords:- Computational experiment, Transportation Problem, Degeneracy Occurrence, Least Cost Method

I. INTRODUCTION

In the Transportation Problem the goal is to minimize the total cost for sending a single commodity from m origins to n destinations subject to offers and demands constraints. Its mathematical formulation, based on [1], [2] and [3] is described as

$$\min = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \tag{1}$$

$$\sum_{j=1}^{n} x_{ij} = a_i, i = 1, \dots, m \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1, \dots, n \tag{3}$$

$$x_{ij} \geq 0, i = 1, \dots, m; j = 1, \dots, n \tag{4}$$

The unitary transportation cost is defined by c_{ij} and the decision variables x_{ij} describe the amount to be shipped from origin i to destination j. The quantity offered by the source i and demanded by destination j are represented respectively by a_i and b_j. The set of constraints (4) indicates that it is not allowed negative amount of transport.

The set of constraints (2) and (3) is composed of $m+n$ equations and there is exactly one redundant constraint [1]. When any one of those constraints is dropped, the remaining system of equation is linearly independent [3]. Therefore, an extreme point is represented by $m+n-1$ basic variables. It implies that when a transportation problem algorithm is implemented it is possible to use a structure with, in the maximum, only $m+n-1$ variables different of zero.

However it is possible that a basic variable assumes value zero, which is called degenerated variable [3]. Depending of the structure used when the MODI algorithm is implemented to solve the transportation problem, degeneracy may be a problem. In this sense this paper aims to study the degeneracy occurrence when the least cost method is used as initial solution for the MODI method.

II. THE MODI METHOD FOR SOLVING TRANSPORTATION PROBLEM

The methods to solve a transportation problem start with an Initial Basic Feasible Solution (IBFS) followed by an iterative improvement procedure. When a basic feasible solution is known, the most referenced method for solution improvement is the MODI.

The MODI method to solve a TP is highly described in the literature, such as Murty [2], Hasan [5] and Loch and Silva [6] and is well known for those having worked with mathematical programming. The method, as cited in Loch and Silva [6] is presented in algorithm 1:

Step1. Obtain an initial basic feasible solution.

Step2. Compute the potential (or multiplier) of each row and column. Using the fact that $c_{ij}=u_i-v_j$ for each basic variable, set $u_1=0$ and, recursively, compute the remaining u_i's and v_j's.

Step3. For each non-basic variable, compute $k_{ij}=c_{ij}-u_i-v_j$.

Step4. If there is a negative k_{ij}, chose the variable x_{pq} associated with the most negative k_{pq} to entry the basis. Otherwise, go to step9.

Step5. Find a *θ-loop* in the set of cells consisted by the cell (p,q) and the basic cells.

Step6. Place an entry of $+\theta$ in the cell (p,q) and alternately the entries of $-\theta$ and $+\theta$ among the cells in the θ-loop. The cells place with $-\theta$ are called donor cell and the ones placed with $+\theta$ are called recipient.

Step7. Identify the donor cell (r,s) with the smallest value (in case of a draw, choose one arbitrarily) and set $\theta=x_{rs}$.

Step8. Compute $x_{ij}=x_{ij}+\theta$ for the recipient cells in the *θ-loop* and $x_{ij}=x_{ij}-\theta$ for the donor cell in the θ-loop. Then cell (r,s) becomes non basic and cell (p,q) becomes basic. Go to Step2.

Step9. Finish. The current solution is optimal.

III. THE LEAST COST METHOD FOR INITIAL FEASIBLE SOLUTION

The Least Cost Method, based on [2] and [4] may be resumed

Step1. Select a variable x_{rs} such as $c_{rs}=\min\{c_{ij}, a'_i>0 \text{ and } b'_j>0\}$

Step2. Set $x_{rs}=\min\{a_r, b_s\}$, $a_r=a_r-x_{rs}$ and $b_s=b_s-x_{rs}$.

Step3. If there is $a_i>0$ or $b_j>0$ return to Step1. Otherwise, Finish.

Numerical examples of the least cost method can be easily found in the literature as in [1], [2] and [3]. So in this paper it is going to be presented in Figure 1 only an example of a solution with degenerated basic variable, without showing how the result was obtained.

	1	2	3	4	5	Supply
1					100	100
2		0	80	0		80
3				120		120
4	60			0		60
5		40			80	120
Demand	60	40	80	120	180	

Figure 1 – Example of solution with three degenerated basic variables

IV. THE COMPUTATIONAL EXPERIMENT

In order to analyze the occurrence of degenerated basic variables in the initial solution when the least cost method is used it was studied transportation problems of different sizes, costs ranges, supply ranges and demand ranges.

The problem sizes studied were 5x5, 10x10, 20x20, 40x40, 80x80, 160x160. The costs (integer values) were generated in different intervals, [3,8], [3,80], [3,800] e [3,8000], and the supply and demand values (integer values) were generated in the intervals [5,50], [5,500], [5,5000] e [5,50000]. Therefore, it was considered 384 parameter combinations and for each one it were created and solved 10000 problems, totaling 3.84 million examples.

The first analysis was about the main effects in the percentage of solved problems with at least one degenerated basic variable. That is, it was identified which factors influence in the percentage of problems with at least one degenerated basic variable in the solution obtained by the Least Cost Method.

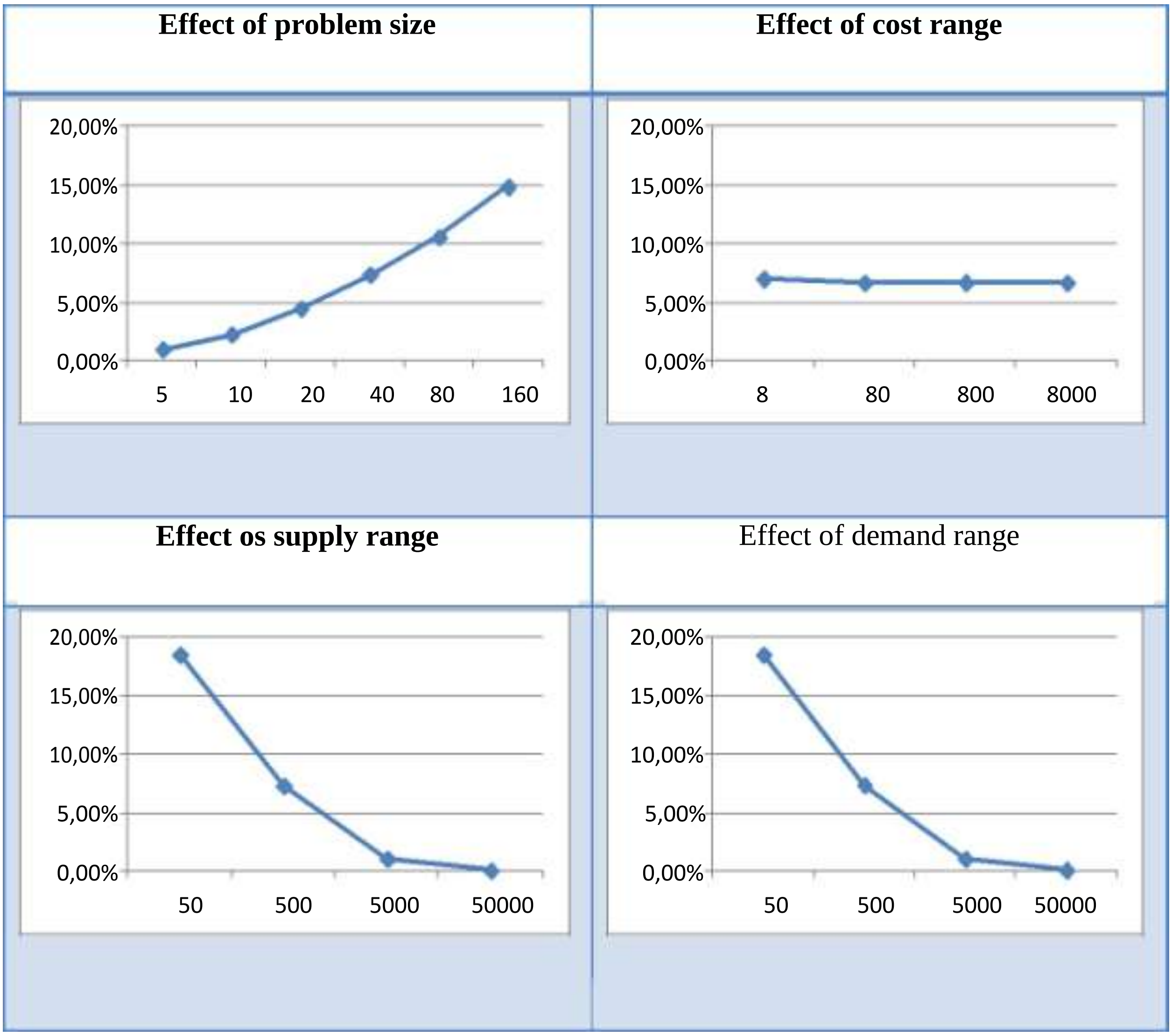

Figure 1 – Main effect analysis of degeneracy occurrence for the least cost method

Based on the graphics of Figure 1 it is possible to conclude, for the studied intervals, that the percentage of problems with degenerated basic variables in the initial solution obtained by the Least Cost Method increases as the problem size increases and decreases as the range of supply and demand increases. Furthermore, there is no influence of costs range.

To complement the graphic analysis and also to obtain information about the interactions among the factor it was computed the ANOVA (exhibit 1) to the computational results.

Exhibit 1 – ANOVA

Source	DF	SS(Aj.)	MS(Aj.)	Fvalue	p-value
ProblemSize	5	0,8910	0,178208	*	*
MaxCusto	3	0,0007	0,000217	*	*
MaxOferta	3	2,0587	0,686249	*	*
MaxDemanda	3	2,0558	0,685274	*	*
ProblemSize *MaxCusto	15	0,0034	0,000225	*	*
ProblemSize *MaxOferta	15	0,7981	0,053204	*	*
ProblemSize *MaxDemanda	15	0,7938	0,052917	*	*
MaxCusto*MaxOferta	9	0,0005	0,000051	*	*
MaxCusto*MaxDemanda	9	0,0005	0,000051	*	*
MaxOferta*MaxDemanda	9	4,7164	0,524050	*	*
ProblemSize *MaxCost*MaxSupply	45	0,0029	0,000065	*	*
ProblemSize *MaxCost*MaxDemand	45	0,0027	0,000060	*	*
ProblemSize *MaxSupply*MaxDemand	45	1,4527	0,032282	*	*
MaxCost*MaxSupply*MaxDemand	27	0,0077	0,000283	*	*
ProblemSize *MaxCost*MaxSupply*MaxDemand	135	0,0136	0,000101	*	*
Error	0	*	*		
Total	383	12,7984			

The sum of squares that presented the least values (less then 0.1) were joined to form the residual and then it was possible to calculate the p-values (Exhibit 2)

Exhibit 2 – ANOVA with residual

Source	DF	SS(Aj.)	MS(Aj.)	Fvalue	p-value
ProblemSize	5	0,891	0,178208	1608,9	4,546E-208
MaxSupply	3	2,0587	0,686249	6195,602	3,588E-261
MaxDemand	3	2,0558	0,685274	6186,8	4,390E-261
ProblemSize *MaxSupply	15	0,7981	0,053204	480,3371	8,343E-194
ProblemSize *MaxDemand	15	0,7938	0,052917	477,746	1,762E-193
MaxOferta*MaxDemand	9	4,7164	0,52405	4731,235	4,224E-307
ProblemSize *MaxSupply*MaxDemand	45	1,4527	0,032282	291,4488	2,269E-214
Residual	288	0,0319	0,000111		
Total	383	12,7984			

When the p-values of Exhibit 2 are analyzed, it is possible to conclude that exists effect of interactions between the factors ProblemSize*MaxSupply, ProblemSize*MaxDemand, MaxSupply*MaxDemand and ProblemSize*MaxSupply*MaxDemand.

For graphical analysis of the existence, or not, of factors interactions, it is observed in each graphic the behavior of the curves. The more parallel lines are, more evidence is there that there is no interaction between factor levels. On the other hand, in the case of non-parallel lines, it is concluded that there is interaction between factor levels.

Based on the graphics of Figure 2 and on Exhibit 1, all second-order interactions involving the cost ranges do not interfere in the percentage of problems that have degenerate basic variables in the initial solution obtained by the least cost method.

On the other hand, when analyzed the interaction between problem size and supply range (MaxSupply), it is observed that the behavior of curves of different supply ranges when the problem size is increased, is not the same. Thus, it is concluded that there is interaction between the problem size and supply range. Analogous situation occurs for the interaction between problem size and demand range.

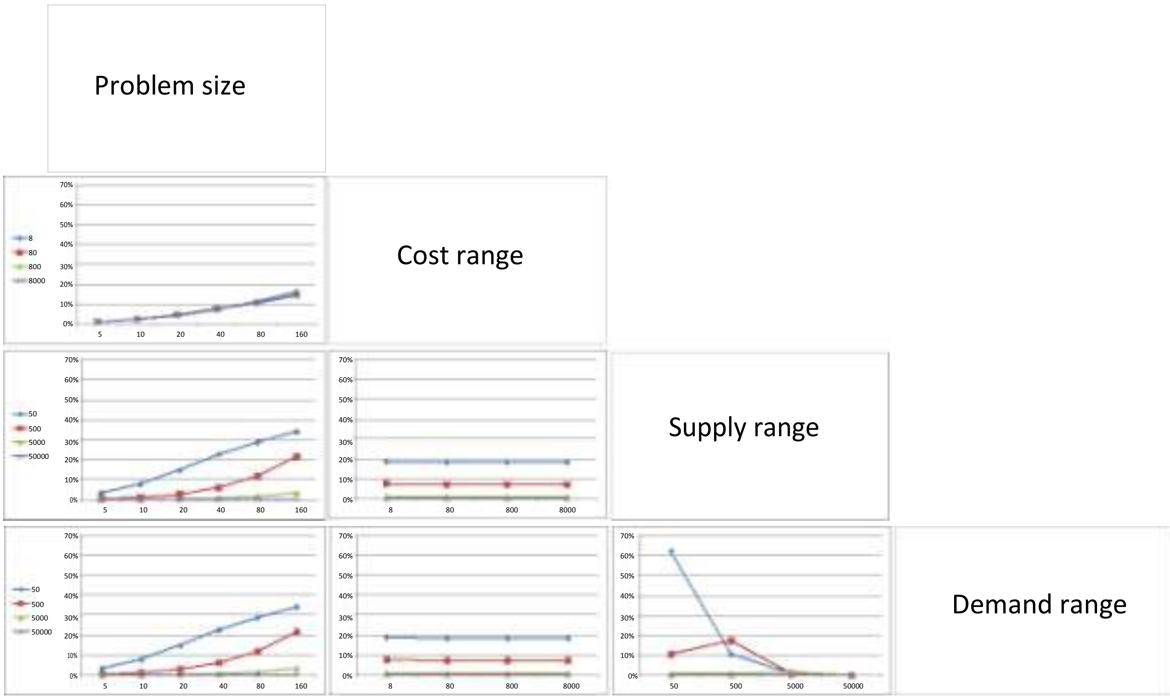

Figure 2 – Graphical analysis of second order interaction between the factors

Based on the results of ANOVA and graphical analysis it is possible to infer that for the parameter levels studied the cost range, as expected, does not influence on the percentage of problems with at least one degenerated basic variable. However, the problem size and the supply and demand ranges interfere on degeneracy.

Finally, for the interaction between the supply and demand ranges, it is noted that for a fixed supply range, when the demand range is equal to supply range, a greater number of problems with degenerate variables occurs after initial solution by the least cost method. These results are set out in Exhibit 3.

Exhibit 3 – Analysis of interaction between Supply and Demand ranges

MaxDemand / MaxSupply	50	500	5000	50000
50	**62,22%**	**10,76%**	**1,00%**	**0,12%**
500	**10,70%**	**17,47%**	**1,19%**	**0,10%**
5000	**1,03%**	**1,20%**	**2,12%**	**0,12%**
50000	**0,09%**	**0,10%**	**0,13%**	**0,23%**

The degeneracy occurred in more than 60% of the problems when the supply and demand ranges were both [5,50], showing that degeneracy is common for small ranges of supply and demand. To complement the analysis it was listed (Exhibit 4) the parameter configurations at which at least 30% of the problems presented degeneracy.

Exhibit 4 – Parameter configurations with at least 30% of the problems presenting degeneracy

Problem size	Cost range	Supply range	Demand range	Problems that presented degeneracy
160	[3,800]	[5,50]	[5,50]	99,95%
160	[3,80]	[5,50]	[5,50]	99,93%
160	[3,8000]	[5,50]	[5,50]	99,90%
160	[3,8]	[5,50]	[5,50]	99,82%
80	[3,800]	[5,50]	[5,50]	97,05%
80	[3,8000]	[5,50]	[5,50]	96,92%

80	[3,80]	[5,50]	[5,50]	96,81%
80	[3,8]	[5,50]	[5,50]	95,40%
40	[3,8000]	[5,50]	[5,50]	81,80%
40	[3,800]	[5,50]	[5,50]	81,67%
40	[3,80]	[5,50]	[5,50]	81,56%
40	[3,8]	[5,50]	[5,50]	78,33%
20	[3,80]	[5,50]	[5,50]	55,23%
20	[3,800]	[5,50]	[5,50]	54,95%
20	[3,8000]	[5,50]	[5,50]	54,78%
20	[3,8]	[5,50]	[5,50]	52,55%
160	[3,800]	[5,500]	[5,500]	50,65%
160	[3,8000]	[5,500]	[5,500]	50,49%
160	[3,80]	[5,500]	[5,500]	49,56%
160	[3,8]	[5,500]	[5,500]	48,05%
160	[3,8]	[5,50]	[5,500]	42,59%
160	[3,8]	[5,500]	[5,50]	42,27%

It is possible to note that large problems with small range of supply and demand present high indices of degeneracy in the initial solution by the least cost method. This kind of result could be expected before this study, but this paper aims to quantify the degeneracy occurrence.

V. CONCLUSION

The degeneracy may occur in transportation problem and depending on the way that the transportation problem is implemented it may cause problems in the algorithm. Because of this it is important a study of degeneracy occurrence. This paper focused on degeneracy when the well known least cost method is used for the initial solution and in quantifies the degeneracy occurrence.

It was observer based on computational experiment with over 3 million problems tested that degeneracy is highly influenced by the supply and demand ranges. Besides that, it was noted that even for larger ranges it is not uncommon. So the structure that is going to be used when the transportation problem algorithm is implemented needs to be prepared to work with degeneracy.

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