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Intuitionistic fuzzy g - closed sets

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Abstract: In this paper, we introduce and study the notions of intuitionistic fuzzy g -closed sets and intuitionistic fuzzy g -open sets and study some of its properties in Intuitionistic fuzzy topological spaces.

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I. INTRODUCTION

In 1965, Zadeh [12] introduced fuzzy sets and in 1968, Chang [2] introduced fuzzy topology. After the introduction of fuzzy set and fuzzy topology, several authors were conducted on the generalization of these notions. The notion of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In 1997, Coker [3] introduced the concept of intuitionistic fuzzy topological spaces. In this paper, we introduce the notions of intuitionistic fuzzy g -closed sets and intuitionistic fuzzy g -open sets and study some of its properties in intuitionistic fuzzy topological spaces.

II. PRELIMINARIES

Throughout this paper, (X; ) or X denotes the intuitionistic fuzzy topological spaces (briefly IFTS). For a subset A of X, the closure, the interior and the complement of A are denoted by cl(A), int(A) and A respectively.

We recall some basic definitions that are used in the sequel.

III. M. THIRUMALAISWAMY

Definition 2.1. [1] Let X be a non-empty fixed set. An intuitionistic fuzzy set A in X is an object having the form

A = f(x; A(x); A(x)): x 2 Xg;

where the functions A : X ! [0; 1] and A : X ! [0; 1] denote the degree of membership A(x) and the degree of non-membership A(x) of each element x 2 X respectively.

Definition 2.2. [1] Let A and B be IFS’s of the form

A = f A(x); A(x)i : x 2 Xg and B = f B(x); B(x)i : x 2 Xg:

Then

1. A B if and only if A(x) and B(x) for all x 2 X:
2. A = B if and only if A and B are:
   1. A(x) = x 2 X:
   2. A(x) = x 2 X:
   3. A(x) = x 2 X:
   4. A(x) = x 2 X:
   5. A(x) = x 2 X:
   6. A(x) = x 2 X:
   7. A(x) = x 2 X:

Denition 2.3. [3] An intuitionistic fuzzy topology (IFT) on X is a family of IFS's in X satisfying the following axioms.

1. G1 G2:
2. G1 G2 for any G1; G2.

In this case the pair (X; ) is called an intuitionistic fuzzy topological space.

www.ijres.com 65 | Page
The complement $A^c$ of an IFOS $A$ in IFTS $(X, )$ is called an intuitionistic fuzzy closed set (IFCS) in $X$.

We simply write $A = hx; A_i$ instead of $A = fhx; A(x); A_i : x 2 Xg$ in case there is no chance for confusion.

Definition 2.4. [3] Let $(X, )$ be an IFTS and $A = hx; A_i$ be an IFS in $X$. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$$S_{\text{int}}(A) = \{G : G \text{ is an IFOS in } X \text{ and } G \supseteq A\}$$

$$cl(A) = \{K : K \text{ is an IFCS in } X \text{ and } A \subseteq K\}$$

Definition 2.5. [5] An IFS $A = hx; A_i$ in an IFTS $(X, )$ is said to be an

1. intuitionistic fuzzy semi-open set (IFSOS) if $A \subseteq cl(int(A));$
2. intuitionistic fuzzy pre-open set (IFPOS) if $A \subseteq int(cl(A));$
3. intuitionistic fuzzy -open set (IF OS) if $A \subseteq int(cl(int(A)));$
4. intuitionistic fuzzy regular open set (IFROS) if $A = int(cl(A));$
5. intuitionistic fuzzy -open set (IF OS) if $A \subseteq cl(int(cl(A)));$

An IFS $A$ is said to be an intuitionistic fuzzy semi-closed set (IFSCS), intuitionistic fuzzy pre-closed set (IFPCS), intuitionistic fuzzy -closed set (IF CS), intuitionistic fuzzy regular closed set (IFRCS) and intuitionistic fuzzy -closed set (IF OS) if the complement of $A$ is an IFSOS, IFPOS, IF OS, IFROS and IF OS respectively.

Definition 2.6. An IFS $A = hx; A_i$ in an IFTS $(X, )$ is said to be an

1. intuitionistic fuzzy generalized closed set (IFGCS) if $cl(A) U$ whenever $A U$ and $U$ is an IFOS in $X,$
2. intuitionistic fuzzy regular generalized closed set (IFRGCS) if $cl(A) U$ whenever $A U$ and $U$ is an IFROS in $X,$
3. intuitionistic fuzzy generalized semi-closed set (IFGSCS) if $scl(A) U$ whenever $A U$ and $U$ is an IFOS in $X,$
4. intuitionistic fuzzy generalized closed set (IFGCS) if $cl(A) U$ whenever $A U$ and $U$ is an IFOS in $X,$
5. intuitionistic fuzzy generalized semi-closed set (IFGSCS) if $spcl(A) U$ whenever $A U$ and $U$ is an IFOS in $X.$
6. intuitionistic fuzzy generalized semipre closed set (IFGSPCS) if $spcl(A) U$ whenever $A U$ and $U$ is an IFOS in $X.$

IV. M. THIRUMALAISWAMY

An IFS $A$ is said to be an intuitionistic fuzzy generalized open set (IFGOS), intuitionistic fuzzy regular generalized open set (IFRGOS), intuitionistic fuzzy generalized semi-open set (IFGOS), intuitionistic fuzzy -generalized open set (IF GOS), intuitionistic fuzzy generalized -open set (IFG OS) and intuitionistic fuzzy generalized semipre open set (IFGSPOS) if the complement of $A$ is an IFGCS, IFRGCS, IFGSCS, IF GCS, IF GCS and IFGSPCS respectively.

Definition 2.7. [11] Let $\alpha; \beta$ be any two functions from $[0, 1]$ with $\alpha + \beta = 1$: An intuitionistic fuzzy point (briefly IFP), written as $p(\alpha; \beta)$, is defined to be an IFS of $X$ given by

$$p(\alpha; \beta)(x) = (\alpha; \beta) \text{ if } x = p, \alpha = (0; 1) \text{ otherwise.}$$

We observe that an IFP $p(\alpha; \beta)$ is said to belong to an IFS $A = hx; A_i; A_i$; denoted by $p(\alpha; \beta) \subseteq A$ if $A(x)$
and $A(x)$:

**Definition 2.8.** [5] Two IFS's $A$ and $B$ are said to be $q$-coincident (brie y $A \sqsubset B$) if and only if there exists an element $x \in X$ such that $A(x) \geq B(x)$ or $A(x) < B(x)$.

**Definition 2.9.** [5] Two IFS's $A$ and $B$ are said to be not $q$-coincident (brie y $A \not\sqsubset B$) if and only if $A \not\sqsubset B$.

**Definition 3.1.** An IFS $A$ of an IFTS $(X; \mu)$ is said to be an intuitionistic fuzzy $g$-closed set (brie y IFG CS) if $\text{cl}(A) \subseteq \text{int}(\text{cl}(U))$ whenever $A \subseteq U$ and $U$ is an IF OS in $(X; \mu)$.

**Example 3.2.** Let $X = \{a, b\}$ and $\mu = \{0, 1\}$ be an IFTS on $X$, where $G = \{h_x, (0.5, 0.6), (0.5, 0.4)\}$: Then the IFS $A = \{h_x, (0.4, 0.5), (0.6, 0.5)\}$ is an IFG CS in $(X; \mu)$.

**Theorem 3.3.** Every IFCS is an IFG CS but not conversely.

**Proof.** Let $A U$; where $U$ is an IFROS. Then $\text{cl}(A) \subseteq A U$: Hence $A$ is an IFG CS.

**Example 3.4.** Let $X = \{a, b\}$ and $\mu = \{0, 1\}$ be an IFTS on $X$, where $G = \{h_x, (0.5, 0.6), (0.5, 0.4)\}$: Then the IFS $A = \{h_x, (0.4, 0.5), (0.6, 0.5)\}$ is an IFG CS but not an IFCS in $(X; \mu)$.

**Theorem 3.5.** Every IFRCS is an IFG CS but not conversely.

**Proof.** Since every IFRCS is an IFCS, the proof follows from Theorem 3.4.

**Example 3.6.** Let $X = \{a, b\}$ and $\mu = \{0, 1\}$ be an IFTS on $X$, where $G = \{h_x, (0.5, 0.6), (0.5, 0.4)\}$: Then the IFS $A = \{h_x, (0.4, 0.5), (0.6, 0.5)\}$ is an IFG CS but not an IFRCS in $(X; \mu)$.

**Theorem 3.7.** Every IF CS is an IFG CS but not conversely.

**Proof.** Let $A$ be an IF CS and $U$ be an IFROS such that $A U$: Then $\text{cl}(A) \subseteq U$: Since $\text{cl}(A) = A$ and hence $A$ is an IFG CS.

**Example 3.8.** Let $X = \{a, b\}$ and $\mu = \{0, 1\}$ be an IFTS on $X$, where $G = \{h_x, (0.5, 0.6), (0.5, 0.4)\}$: Then the IFS $A = \{h_x, (0.4, 0.5), (0.6, 0.5)\}$ is an IFG CS but not an IF CS in $(X; \mu)$.

**Theorem 3.9.** Every IFGCS is an IFG CS but not conversely.

**Proof.** Let $A$ be an IFGCS and $U$ be an IFROS such that $A U$: Since every IFROS is an IFOS and $\text{cl}(A)$ is an IFOS in $(X; \mu)$, we have by hypothesis, $\text{cl}(A) \subseteq \text{cl}(U)$ and hence $A$ is an IFG CS.

**Example 3.10.** Let $X = \{a, b\}$ and $\mu = \{0, 1\}$ be an IFTS on $X$, where $G = \{h_x, (0.5, 0.6), (0.5, 0.4)\}$: Then the IFS $A = \{h_x, (0.4, 0.5), (0.6, 0.5)\}$ is an IFG CS but not an IFGCS in $(X; \mu)$.

**Theorem 3.11.** Every IFRGCS is an IFG CS but not conversely.

**Proof.** Let $A$ be an IFRGCS and $U$ be an IFROS such that $A U$: Since $\text{cl}(A) \subseteq \text{cl}(U)$ and $\text{cl}(A) U$; by hypothesis, $A$ is an IF CS.

**Example 3.12.** Let $X = \{a, b, c\}$ and $\mu = \{0, 1\}$ be an IFTS on $X$, where $G_1 = \{h_x, (0.5, 0.6), (0.5, 0.4)\}$: Then the IFS $A = \{h_x, (0.4, 0.5), (0.6, 0.5)\}$ is an IFG CS but not an IFRCS in $(X; \mu)$.
Intuitionistic fuzzy $g$-closed sets

$G_1 = \{x; (0.4; 0.4; 0.5); (0.4; 0.4; 0.4)\}$ and $G_2 = \{x; (0.2; 0.3; 0.5); (0.5; 0.5; 0.5)\}$:

Then the IFS $A = \{x; (0.4; 0.3; 0.2); (0.5; 0.4; 0.5)\}$ is an IFG CS but not an IFRGCS in $(X; )$.

**Theorem 3.13. Every IF GCS is an IFG CS but not conversely.**

Proof. Let $A$ be an IF GCS and $U$ be an IFROS such that $A \cup U$. Since every IFROS is an IFOS and $A$ is an IF GCS, we have $cl(A) \cup U$. Hence $A$ is an IFG CS.

**Example 3.14.** Let $X = \{a; b\}$ and $\mathcal{F} = \{0, ~ G; 1\}$ be an IFTS on $X$, where $G = \{x; (0.5; 0.6); (0.5; 0.4)\}$: Then the IFS $A = \{x; (0.4; 0.5); (0.6; 0.5)\}$ is an IFG CS but not an IF GCS in $(X; )$.

**Theorem 3.15. Every IFG CS is an IFG CS but not conversely.**

Proof. Let $A$ be an IFG CS and $U$ be an IFROS such that $A \cup U$. Since every IFROS is an IF OS and by hypothesis, we have $cl(A) \cup U$. Hence $A$ is an IFG CS.

**Example 3.16.** Let $X = \{a; b\}$ and $\mathcal{F} = \{0, ~ G; 1\}$ be an IFTS on $X$, where $G = \{x; (0.8; 0.8); (0.2; 0.1)\}$: Then the IFS $A = \{x; (0.9; 0.7); (0.1; 0.3)\}$ is an IFG CS but not an IFG CS in $(X; )$.

**Remark 3.17.** Summing up the above theorems, we have the following diagram. None of the implications are reversible.

<table>
<thead>
<tr>
<th>IFCS</th>
<th>! IFGCS</th>
<th>! IFRGCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>&amp;</td>
<td>IFG CS</td>
<td></td>
</tr>
<tr>
<td>&amp;</td>
<td></td>
<td>! IF GCS</td>
</tr>
</tbody>
</table>

**Remark 3.18.** The following examples show that IFG CS is independent of IFPCS, IFSCS, IF CS, IFGSCS and IFGSPCS.

**Example 3.19.** Let $X = \{a; b\}$ and $\mathcal{F} = \{0, ~ G_1; G_2; 1\}$ be an IFTS on $X$, where

$G_1 = \{x; (0.7; 0.8); (0.3; 0.2)\}$ and $G_2 = \{x; (0.6; 0.7); (0.4; 0.3)\}$: Then the IFS $A = \{x; (0.6; 0.8); (0.4; 0.2)\}$ is an IFG CS but not an IFPCS, IFSCS, IF CS, IFRGCS and IFGSPCS in $(X; )$.

**Example 3.20.** Let $X = \{a; b\}$ and $\mathcal{F} = \{0, ~ G; 1\}$ be an IFTS on $X$, where $G = \{x; (0.5; 0.4); (0.5; 0.6)\}$: Then the IFS $A = \{x; (0.4; 0.2); (0.6; 0.7)\}$ is an IFPCS, IF CS, IFGSCS.
Intuitionistic fuzzy \(g\)-closed sets

and IFGSPCS but not an IFG CS in \((X; g)\):

**Example 3.21.** Let \(X = a; b\) and \(= f_0; G; 1-g\) be an IFTS on \(X\), where

\[ G = hx; (0.5; 0.4); (0.5; 0.6)i : \]

Then the IFS \(A = hx; (0.5; 0.5); (0.5; 0.6)i\) is an IFSCS but not an IFG CS in \((X; g)\):

**Theorem 3.22.** The union of two IFG C sets is an IFG CS in \((X; g)\):

**Proof.** Let \(U\) be an IFROS in \((X; g)\) such that \(A[B U\): Then \(A U\) and \(B U\): So, \(cl(A) U\) and \(cl(B) U\):

Therefore \(cl(A) \cap cl(B) cl(A B) U\):

Hence \(A B\) is an IFG CS.

**Remark 3.23.** Intersection of two IFG C sets need not be an IFG CS.

**Example 3.24.** Let \(X = a; b; c\) be an IFTS on \(X\), where \(G_1 = hx; (0.0; 0.6; 0.1); (0.1; 0.25; 0.0)i \) and \(G_2 = hx; (0.1; 0.25; 0.0); (0.0; 0.6; 0.1)i\):

Then the IFS \(A = hx; (0.3; 0.7; 0.1); (0.2; 0.25; 0.3)i\) and the IFS \(B = hx; (0.0; 0.6; 0.3); (0.1; 0.3; 0.0)i\) are IFG C sets but \(A B\) is not an IFG CS.

**Theorem 3.25.** If an IFS \(A\) is an IFG CS such that \(A \cap cl(A)\) where \(B\) is an IFS in an IFTS \((X; g)\):

**Proof.** Let \(U\) be an IFROS in \((X; g)\) such that \(B U\): Then \(A U\):

Since \(A\) is an IFG CS, we have \(cl(A) U\): Now, \(cl(B) cl(cl(A)) = cl(A) U:\)

Hence \(B\) is an IFG CS in \((X; g)\):

**Theorem 3.26.** If an IFS \(A\) is an IFRGCs such that \(A B cl(A)\); where \(B\) is an IFS in an IFTS \((X; g)\); then \(B\) is an IFG CS in \((X; g)\):

**Proof.** Let \(U\) be an IFROS in \((X; g)\) such that \(B U\): Then \(A U\): Since \(A\) is an IFRGCs and \(cl(A) cl(A)\); we have \(cl(A) cl(A) U:\)

**Theorem 3.27.** An IFS \(A\) is an IFG CS in an IFTS \((X; g)\) if and only if \(e(AqF )\) implies \(e(cl(A)qF )\) for every IFRCS \(F\) of \((X; g)\):

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Proof. Necessity. Assume that $A$ is an IFG CS in $(X; \cdot)$: Let $F$ be an IFRCS and $e(AqF)$: Then $A \subseteq F^c$; where $F^c$ is an IFROS in $(X; \cdot)$: Then by assumption,

$$\text{cl}(A) \supseteq F^c$$

Hence $e(\text{cl}(A)qF)$:

Sufficiency. Let $F$ be an IFROS in $(X; \cdot)$ such that $F \subseteq U$: Then $F^c$ is an IFRCS in $(X; \cdot)$ and $F \subseteq (U^c)^c$: By assumption, $e(FqU^c)$ implies $e(\text{cl}(A)qU^c)$:

Therefore, $\text{cl}(A) \supseteq (U^c)^c = U$: Hence $A$ is an IFG CS in $(X; \cdot)$:

Theorem 3.28. If $A$ is an IFROS and an IFG CS in $(X; \cdot)$: then $A$ is an IF CS in $(X; \cdot)$:

Proof. Let $A$ be an IFROS. Since $A \subseteq A$: $\text{cl}(A) \subseteq A$: Therefore $\text{cl}(A) = A$: Hence $A$ is an IF CS in $(X; \cdot)$:

Theorem 3.29. Every IFS in an $(X; \cdot)$ is an IFG CS if and only if IF OS and IF CS coincide.

Proof. Necessity. Suppose that every IFS in $(X; \cdot)$ is an IFG CS. Let $U$ be an IFROS in $(X; \cdot)$: Then $U$ is an IFOS and an IF OS and by hypothesis

$$\text{cl}(U) = U$$

Hence IF O(X) IF C(X): Let $A$ be an IF CS. Then $A^c$ is an IF OS in $(X; \cdot)$: But IF O(X) IF C(X): Therefore $A$ is an IF OS in $(X; \cdot)$: we have

$$\text{IF C(X) IF O(X): Thus IF O(X) = IF C(X):}$$

Sufficiency. Suppose that IF O(X) = IF C(X): Let $A \subseteq U$ and $U$ be an IFROS in $(X; \cdot)$: Since every IFROS is IF OS, $U$ is an IF OS in $(X; \cdot)$ and therefore $\text{cl}(A) \subseteq U$: by hypothesis. Hence $A$ is an IFG CS in $(X; \cdot)$:

Theorem 3.30. An IFS $A$ of an IFTS $(X; \cdot)$ is an IFROS and an IFG CS, then $A$ is an IFRCS in $(X; \cdot)$:

<table>
<thead>
<tr>
<th>Proof. Let $A$ be an IFROS and an IFG CS in $(X; \cdot)$: Then</th>
<th>$\text{cl}(A)$</th>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{cl}(A)$ is an IF CS, we have $\text{cl}(\text{int}(\text{cl}(A)))$</td>
<td>$A$: Therefore $\text{cl}(A)$</td>
<td>$A$:</td>
</tr>
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</table>

since $A$ is an IFROS. Then $\text{cl}(\text{int}(A))$ $\text{cl}(A)$ $A$: Therefore $\text{cl}(\text{int}(A))$ $A$: |
Since every IFROS is an IFSOS, A is an IFSOS and we have $A = \text{cl} (\text{int}(A))$: Thus

$A = \text{cl} (\text{int}(A))$: Hence A is an IFRCS in $(X; )$.

Theorem 3.31. Let A be an IFG CS in $(X; )$ and $p_1(\cdot)$ be an IFP in X such that $\text{cl}(A) \leq \text{cl}(p_1(\cdot))$: Then $A \leq \text{cl}(p_1(\cdot))$

Proof. Assume that A is an IFG CS in $(X; )$ and $\text{cl}(A) \leq \text{cl}(p_1(\cdot))$: Suppose that $e \notin (\text{cl}(A) \leq \text{cl}(p_1(\cdot)))$: then $A \leq (\text{cl}(p_1(\cdot)))^c$ where $(\text{cl}(p_1(\cdot)))^c$ is an IF OS in $(X; )$: Then by Definition 3.1, $\text{cl}(A) \leq \text{int}(\text{cl}(\text{cl}(p_1(\cdot)))^c) \leq \text{cl}(\text{cl}(p_1(\cdot)))^c$

Definition 4.1. An IFS A of an IFTS $(X; )$ is called an IFG OS if and only if $A^c$ is an IFG CS.

Theorem 4.2. Every IFOS, IFROS, IFG OS is an IFG CS in $(X; )$: IF OS, IFGOS, IFRGOS, IF GOS,

IFG OS is an IFG OS in $(X; )$: Proof. Obvious.

Example 4.3. Let $X = \{a, b\}$ and $G = \{0, 0.5; 0.6; 0.5; 0.4\}$: Then the IFS $A = \{0.6; 0.5; 0.4\}$ is an IFG OS but not an IFOS, IFROS, IF OS, IFGOS, IF GOS in $(X; )$:

Example 4.4. Let $X = \{a, b\}$ and $G = \{0.8; 0.8; 0.2; 0.1\}$: Then the IFS $A = \{0.1; 0.3; 0.9; 0.7\}$ is an IFG OS but not an IFG OS in $(X; )$:

Example 4.5. Let $X = \{a, b\}$ and $G_1 = \{0.4; 0.4\}$ and $G_2 = \{0.5; 0.4; 0.4\}$: Then the IFS $A = \{0.5; 0.4; 0.4\}$ is an IFG CS but not an IFRGCS in $(X; )$:
Theorem 4.6. An IFS $A$ of an IFTS $(X; ;)$ is an IFG OS if and only if

$U \cap \text{int}(A)$ whenever $U \subseteq A$ and $U$ is an IFRCS.

Proof. Necessity. Assume that $A$ is an IFG OS in $(X; ;)$. Let $U$ be an IFRCS such that $U \cap A$. Then $U^c$ is an IFROS and $A^c \cap U^c$. Then by assumption $A^c$ is an IFG CS in $(X; ;)$. Therefore, we have $\text{cl}(A^c) \subseteq U^c$. Hence $U \cap \text{int}(A)$:

Sufficiency. Let $U$ be an IFROS in $(X; ;)$ such that $A^c \subseteq U$. Then $U^c \cap A^c$. Therefore $U^c$ is an IFRCS. Hence $U^c \cap \text{int}(A)$; we have $A^c$ is an IFG CS. Hence $A$ is an IFG OS in $(X; ;)$.

Remark 4.7. Intersection of two IFG O sets is an IFG OS in $(X; ;)$. But the union of two IFG O sets need not be an IFG OS.

Example 4.8. Let $X = \{a; b; c\}$ be an IFTSon $X$, where $G_1 = \{ (0; 0; 0; 0); (0; 0; 0; 0) \}$ and $G_2 = \{ (0; 0; 0; 0); (0; 0; 0; 0) \}$. Then the IFS $A = \{ (0; 0; 0; 0); (0; 0; 0; 0) \}$ are IFG O sets but $A \cap B$ is not an IFG OS.

Theorem 4.9. Let $A$ be an IFS in $(X; ;)$ if $B$ is an IFS OS such that $B \subseteq \text{int}(\text{cl}(B))$, then $A$ is an IFG OS in $(X; ;)$.

Proof. Since $B$ is an IFSOS, we have $B \subseteq \text{cl}(\text{int}(B))$. Thus, $A \subseteq \text{int}(\text{cl}(B)) \subseteq \text{int}(\text{cl}(\text{int}(B))) = \text{int}(\text{cl}(\text{int}(B)))$. This implies $A$ is an IF OS. By Theorem 4.2, $A$ is an IFG OS in $(X; ;)$.

Theorem 4.10. If an IFS $A$ is an IFG OS in $(X; ;)$ such that $\text{int}(A)$

$A$; where $B$ is an IFS in $(X; ;)$ then $A$ is an IFG OS in $(X; ;)$.

Proof. Suppose that $A$ is an IFG OS in $(X; ;)$, and $\text{int}(A)$

$A^c$ is an IFG CS and $A^c \subseteq \text{int}(A)^c$; this implies $A^c$

Then $B^c$ is an IFG CS in $(X; ;)$ by Theorem 3.26. Hence $B$ is an IFG OS in $(X; ;)$.
Theorem 4.11. If an IFS A is an IFRGOS in (X; ) such that \( \text{int}(A) \subseteq B \subseteq A \); where B is an IFS in (X; ); then B is an IFG OS in (X; ).

Proof. Let A be an IFRGOS and \( \text{int}(A) \subseteq B \subseteq A \); Then \( A^c \) is an IFRGCS and \( A^c \subseteq B^c \subseteq \text{cl}(A^c) \); Then \( B^c \) is an IFG CS in (X; ); by Theorem 3.27. Hence B is an IFG OS in (X; ).

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