Abstract: In this paper, we introduce and study the notions of intuitionistic fuzzy $g^{**}$-closed sets and intuitionistic fuzzy $g^{**}$-open sets and study some of its properties in Intuitionistic fuzzy topological spaces.

Keywords And Phrases: Intuitionistic fuzzy topology, Intuitionistic fuzzy $g^{**}$-closed sets and Intuitionistic fuzzy $g^{**}$-open sets.

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1 Introduction

In 1965, Zadeh [13] introduced fuzzy sets and in 1968, Chang [2] introduced fuzzy topology. After the introduction of fuzzy set and fuzzy topology, several authors were conducted on the generalization of this notions. The notion of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In 1997, Coker [3] introduced the concept of intuitionistic fuzzy topological spaces. In this paper, we introduce the notions of intuitionistic fuzzy $g^{**}$-closed sets and intuitionistic fuzzy $g^{**}$-open sets and study some of its properties in intuitionistic fuzzy topological spaces.

2 Preliminaries

Throughout this paper, $(X, \tau)$ or $X$ denotes the intuitionistic fuzzy topological spaces (briefly IFTS). For a subset $A$ of $X$, the closure, the interior and the complement of $A$ are denoted by $cl(A)$, $int(A)$ and $A^c$ respectively. We recall some basic definitions that are used in the sequel.

Definition 2.1: [1] Let $X$ be a non-empty fixed set. An intuitionistic fuzzy set (IFS in short) $A$ in $X$ is an object having the form $A = \{(x, \mu_A(x), \nu_A(x))/ x \in X\}$ where the functions $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set $A$ respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in $X$.

Definition 2.2: [1] Let $A$ and $B$ be IFS's of the form $A = \{(x, \mu_A(x), \nu_A(x))/ x \in X\}$ and $B = \{(x, \mu_B(x), \nu_B(x))/ x \in X\}$. Then
\begin{enumerate}
  1. $A \subseteq B$ if and only if $\mu_A(X) \leq \mu_B(X)$ and $\nu_A(X) \geq \nu_B(x)$ for all $x \in X$.
  2. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
  3. $A^c = \{ (x, \nu_A(x), \mu_A(x))/ x \in X\}$.
  4. $A \cap B = \{ (x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x))/ x \in X\}$.
  5. $A \cup B = \{ (x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x))/ x \in X\}$.
\end{enumerate}

For the sake of simplicity, we shall use the notation $A = (x, \mu_A, \nu_A)$ instead of $A = \{(x, \mu_A(x), \nu_A(x))/ x \in X\}$. Also for the sake of simplicity, we shall use the notation $A = (x, (\mu_A, \mu_B), (\nu_A, \nu_B))$ instead of $A = (x, (\mu_A, \mu_B), (\nu_A, \nu_B))$. The intuitionistic fuzzy sets $0_1 = \{(x, 0, 1)/ x \in X\}$ and $1_1 = \{(x, 1, 0)/ x \in X\}$ are respectively the empty set and the whole set of $X$.

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on $X$ is a family $\tau$ of IFSs in $X$ satisfying the following axioms:
\begin{enumerate}
  1. $0_1, 1_1 \in \tau$.
  2. $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$.
  3. $\cup G_i \in \tau$ for any family $\{G_i/ i \in J\} \subseteq \tau$.
\end{enumerate}

In this case the pair $(X, \tau)$ is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in $\tau$ is known as an intuitionistic fuzzy open set(IFOS in short) in $X$. The complement $A^c$ of an IFOS $A$ in an IFTS $(X, \tau)$ is called an intuitionistic fuzzy closed set (IFCS in short) in $X$. 

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**Definition 2.4:** Let $(X, \tau)$ be an IFTS and $A = (x, \mu_A, \nu_A)$ be an IFS in $X$. Then

1. $\text{int}(A) = \bigcup \{ G / G$ is an IFOS in $X$ and $G \subseteq A \}$,
2. $\text{cl}(A) = \bigcap \{ K / K$ is an IFCS in $X$ and $A \subseteq K \}$,
3. $\text{cl}(A^\alpha) = (\text{int}(A))^\beta$,
4. $\text{int}(A^\alpha) = (\text{cl}(A))^\beta$.

**Definition 2.5:** An IFS $A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \}$ in an IFTS $(X, \tau)$ is said to an

1. intuitionistic fuzzy semi-open set (IFSOS) if $A \subseteq \text{cl}(\text{int}(A))$,
2. intuitionistic fuzzy pre-open set (IFP) if $A \subseteq \text{int}(\text{cl}(A))$,
3. intuitionistic fuzzy $\alpha$-open set (IFαOS) if $A \subseteq \text{int}(\text{cl}(A))$,
4. intuitionistic fuzzy regular open set (IFROS) if $A = \text{cl}(\text{int}(A))$,
5. intuitionistic fuzzy $\beta$-open set (IFβOS) if $A \subseteq \text{cl}(\text{int}(A))$,
6. intuitionistic fuzzy $\alpha$-open set (IFααOS) if there exist an IFROS $U$ such that $U \subseteq A \subseteq \text{acl}(U)$.

An IFS $A$ is said to be an intuitionistic fuzzy semi-closed set (IFSCS), intuitionistic fuzzy pre-closed set (IFPCS), intuitionistic fuzzy $\alpha$-closed set (IFαCS), intuitionistic fuzzy regular closed set (IFRGCS) and intuitionistic fuzzy $\beta$-closed set (IFβCS) if the complement of $A$ is an IFSCS, IFPCS, IFROS, IFβOS and IFβOS respectively.

**Definition 2.6:** An IFS $A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \}$ in an IFTS $(X, \tau)$ is said to an

1. intuitionistic fuzzy generalized closed set (IFGCS) [10] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFOS in $X$,
2. intuitionistic fuzzy regular generalized closed set (IFRGCS) [9] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFROS in $X$,
3. intuitionistic fuzzy generalized semi-closed set (IFGSCS) [7] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFOS in $X$,
4. intuitionistic fuzzy $\alpha$-generalized closed set (IFαGCS) [6] if $\text{acl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFSCS in $X$,
5. intuitionistic fuzzy generalized $\alpha$-closed set (IFαGCS) [8] if $\text{acl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFβOS in $X$,
6. intuitionistic fuzzy generalized semi-pre closed set (IFGSPCS) [5] if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFOS in $X$,
7. intuitionistic fuzzy regular generalized $\alpha$-closed set (IFRGαCS) [12] if $\text{acl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFRGαOS in $X$.

An IFS $A$ is said to be an intuitionistic fuzzy generalized open set (IFGOS), intuitionistic fuzzy regular generalized open set (IFRGOS), intuitionistic fuzzy regular generalized $\alpha$-open set (IFGαOS), intuitionistic fuzzy generalized semi-open set (IFGSOS), intuitionistic fuzzy $\alpha$-generalized open set (IFαGOS), intuitionistic fuzzy regular generalized $\alpha$-open set (IFRGαOS) and intuitionistic fuzzy generalized semi-pre open set (IFGSPCS) if the complement of $A$ is an IFGCS, IFRGCS, IFGαCS, IFGSCS, IFαGCS, IFRGαCS and IFGSPCS respectively.

**Definition 2.7:** Let $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (briefly IFP), written as $p_{(\alpha, \beta)}$, is defined to be an IFS of $X$ given by

$$p_{(\alpha, \beta)}(X) = \begin{cases} (\alpha, \beta), & \text{if } x = P, \\ (0, 1), & \text{otherwise}. \end{cases}$$

We observe that an IFP $p_{(\alpha, \beta)}$ is said to belong to an IFS $A = (x, \mu_A(x), \nu_A(x))$, denoted by $p_{(\alpha, \beta)} \in A$ if $\alpha \leq \mu_A(x)$ and $\beta \geq \nu_A(x)$.

**Definition 2.8:** [4] Two IFSs $A$ and $B$ are said to be $q$-coincident ($A \sim B$ in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ and $\nu_A(x) < \mu_B(x)$.

**Definition 2.9:** [4] Two IFSs are said to be not $q$-coincident ($A \nparallel B$ in short) if and only if $A \subseteq B^*$.

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**Definition 3.1:** An IFS $A$ of an IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy $g$ $\alpha$**-closed set (briefly IFGαCS) if $\text{acl}(A) \subseteq \text{int}(\text{cl}(U))$ whenever $A \subseteq U$ and $U$ is an IFOS in $(X, \tau)$. [i.e., if $\alpha \text{ cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFROS in $(X, \tau)$]
Example 3.2: Let $X = \{a, b\}$ and $\tau = \{0, G, 1\}$ be an IFTS on $X$, where $G = (x, (0.5, 0.6), (0.5, 0.4))$. Then the IFS $A = (x, (0.4,0.5), (0.6,0.5))$ is an IFG $\alpha**$CS in $(X, \tau)$.

Theorem 3.3: Every IFCS is an IFG$\alpha**$CS but not conversely.

Proof: Let $A \subseteq U$, where $U$ is an IFROS. Then $acl(A) \subseteq cl(A) = A \subseteq U$. Hence $A$ is an IFG$\alpha**$CS.

Example 3.4: Let $X = \{a, b\}$ and $\tau = \{0, G, 1\}$ be an IFTS on $X$, where $G = (x, (0.5, 0.6), (0.5, 0.4))$. Then the IFS $A = (x, (0.4,0.5), (0.6,0.5))$ is an IFG$\alpha**$CS but not an IFCS in $(X, \tau)$.

Theorem 3.5: Every IFRCS is an IFG$\alpha**$CS but not conversely.

Proof: Since every IFRCS is an IFCS, the proof follows from Theorem 3.3.

Example 3.6: Let $X = \{a, b\}$ and $\tau = \{0, G, 1\}$ be an IFTS on $X$, where $G = (x, (0.5, 0.6), (0.5, 0.4))$. Then the IFS $A = (x, (0.4,0.5), (0.6,0.5))$ is an IFG$\alpha**$CS but not an IFRCS in $(X, \tau)$.

Theorem 3.7: Every IF$\alpha$CS is an IFG$\alpha**$CS but not conversely.

Proof: Let $A$ be an IF$\alpha$CS and $U$ be an IFROS such that $A \subseteq U$. Then $acl(A) \subseteq U$. Since $acl(A) = A$ and hence $A$ is an IFG$\alpha**$CS.

Example 3.8: Let $X = \{a, b\}$ and $\tau = \{0, G, 1\}$ be an IFTS on $X$, where $G = (x, (0.5, 0.6), (0.5, 0.4))$. Then the IFS $A = (x, (0.4,0.5), (0.6,0.5))$ is an IFG$\alpha**$CS but not an IF$\alpha$CS in $(X, \tau)$.

Theorem 3.9: Every IFG$\alpha$CS is an IFG$\alpha**$CS but not conversely.

Proof: Let $A$ be an IFG$\alpha$CS and $U$ be an IFROS such that $A \subseteq U$. Since every IFROS is an IFOS and $acl(A) \subseteq cl(A)$, we have $acl(A) \subseteq cl(A) \subseteq U$ and hence $A$ is an IFG$\alpha**$CS.

Example 3.10: Let $X = \{a, b\}$ and $\tau = \{0, G, 1\}$ be an IFTS on $X$, where $G = (x, (0.5, 0.6), (0.5, 0.4))$. Then the IFS $A = (x, (0.4,0.5), (0.6,0.5))$ is an IFG$\alpha**$CS but not an IFG$\alpha$CS in $(X, \tau)$.

Theorem 3.11: Every IFG$\alpha$GCS is an IFG$\alpha**$CS but not conversely.

Proof: Let $A$ be an IFG$\alpha$GCS and $U$ be an IFROS such that $A \subseteq U$. Since every IFROS is an IFROS and $acl(A) \subseteq cl(A)$, by hypothesis we have $acl(A) \subseteq cl(A) \subseteq U$ and hence $A$ is an IFG$\alpha**$CS.

Example 3.12: Let $X = \{a, b, c\}$ and $\tau = \{0, G_1, G_2, 1\}$ be an IFTS on $X$, where $G_1 = (x, (0.4, 0.4, 0.5), (0.4, 0.4, 0.4))$ and $G_2 = (x, (0.2, 0.3, 0.5), (0.5, 0.5, 0.5))$. Then the IFS $A = (x, (0.4,0.3, 0.2), (0.5, 0.4, 0.5))$ is an IFG$\alpha**$CS but not an IFG$\alpha$GCS in $(X, \tau)$.

Theorem 3.13: Every IFG$\alpha$GCS is an IFG$\alpha**$CS but not conversely.

Proof: Let $A$ be an IFG$\alpha$GCS and $U$ be an IFROS such that $A \subseteq U$. Since every IFROS is an IFOS and $acl(A) \subseteq cl(A)$ and $cl(A) \subseteq U$, By hypothesis, $A$ is an IFG$\alpha**$CS.

Example 3.14: Let $X = \{a, b, c\}$ and $\tau = \{0, G_1, G_2, 1\}$ be an IFTS on $X$, where $G_1 = (x, (0.4,0.4, 0.5), (0.4,0.4,0.4))$ and $G_2 = (x, (0.2, 0.3, 0.5), (0.5, 0.5, 0.5))$. Then the IFS $A = (x, (0.4,0.3, 0.2), (0.5, 0.4, 0.5))$ is an IFG$\alpha**$CS but not an IFG$\alpha$GCS in $(X, \tau)$.

Theorem 3.15: Every IF$\alpha$GCS is an IFG$\alpha**$CS but not conversely.

Proof: Let $A$ be an IF$\alpha$GCS and $U$ be an IFROS such that $A \subseteq U$. Since every IFROS is an IFOS and $A$ is IFG$\alpha$CS, we have $acl(A) \subseteq U$. Hence $A$ is an IFG$\alpha**$CS.

Example 3.16: Let $X = \{a, b\}$ and $\tau = \{0, G, 1\}$ be an IFTS on $X$, where $G = (x, (0.5, 0.6), (0.5, 0.4))$. Then the IFS $A = (x, (0.4,0.5), (0.6,0.5))$ is an IFG$\alpha**$CS but not an IF$\alpha$GCS in $(X, \tau)$.
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Theorem 3.17: Every IFGαCS is an IFGα** CS but not conversely.

Proof: Let A be an IFGαCS and U be an IFROS such that A ⊆ U. Since every IFROS is an IFαOS and by hypothesis, we have αcl(A) ⊆ U. Hence A is an IFGα** CS.

Example 3.18: Let X = {a, b} and U = {0, G, 1} be an IFTS on X, where \(G = (x, (0.8, 0.8), (0.2, 0.1))\). Then the IFS \(A = (x, (0.9, 0.7), (0.1, 0.3))\) is an IFGα** CS but not an IFGαCS in \((X, \tau)\).

Remark 3.19: Summing up the above theorems, we have the following diagram. None of the implications are reversible.

\[\text{IFRGαCS} \quad \xrightarrow{\text{IFCS}} \quad \text{IFGCS} \quad \xrightarrow{\text{IFRCS}} \quad \text{IFGα**CS}\]

\[\text{IFRCS} \quad \xrightarrow{\text{IFFaCS}} \quad \text{IFGαCS} \quad \xrightarrow{\text{IFFaCS}} \quad \text{IFFaCS}\]

Remark 3.20: The following examples show that IFGα** CS is independent of IFPCS, IFSCS, IFβCS, IFGSCS and IFGSPCS.

Example 3.21: Let X = {a, b} and \(\tau = \{0, G_1, G_2, 1\}\) be an IFTS on X, where \(G_1 = (x, (0.7, 0.8), (0.3, 0.2))\) and \(G_2 = (x, (0.6, 0.7), (0.4, 0.3))\). Then the IFS \(A = (x, (0.6, 0.8), (0.4, 0.2))\) is an IFGα** CS but not an IFPCS, IFFaCS, IFFaCS and IFGSPCS in \((X, \tau)\).

Example 3.22: Let X = {a, b} and \(\tau = \{0, G, 1\}\) be an IFTS on X, where \(G = (x, (0.5, 0.4), (0.5, 0.6))\). Then the IFS \(A = (x, (0.4, 0.2), (0.6, 0.7))\) is an IFPCS, IFFaCS, IFFaCS and IFGSPCS but not an IFGα** CS in \((X, \tau)\).

Example 3.23: Let X = {a, b} and \(\tau = \{0, G, 1\}\) be an IFTS on X, where \(G = (x, (0.5, 0.5), (0.5, 0.6))\). Then the IFS \(A = (x, (0.5, 0.5), (0.5, 0.6))\) is an IFFaCS but not an IFGα** CS in \((X, \tau)\).

Theorem 3.24: The union of two IFGα** CS sets is an IFGα** CS in \((X, \tau)\).

Proof: Let U be an IFROS in \((X, \tau)\) such that \(A \cup B \subseteq U\). Then \(A \subseteq U\) and \(B \subseteq U\). Hence \(A \cup B\) is an IFGα** CS.

Remark 3.25: Intersection of two IFGα** CS sets need not be an IFGα** CS.

Example 3.26: Let X = {a, b, c} and \(\tau = \{0, G_1, G_2, G_3, G_4\}\) be an IFTS on X, where \(G_1 = (x, (0.0, 0.6, 0.1), (0.1, 0.25, 0.0), (0.0, 0.6, 0.1))\) and \(G_2 = (x, (0.3, 0.7, 0.1), (0.2, 0.25, 0.3))\) and the IFS \(B = (x, (0.3, 0.6, 0.3), (0.1, 0.3, 0.0))\) are IFGα** CS sets but \(A \cap B\) is not an IFGα** CS.

Theorem 3.27: If an IFS A is an IFGα** CS such that \(A \subseteq B \subseteq \text{acl}(A)\), where B is an IFS in an IFTS \((X, \tau)\), then B is an IFGα** CS in \((X, \tau)\).

Proof: Let U be an IFROS in \((X, \tau)\) such that \(B \subseteq U\). Then \(A \subseteq U\). Since A is an IFGα** CS, we have \(\text{acl}(A) \subseteq U\). Hence B is an IFGα** CS in \((X, \tau)\).

Theorem 3.28: If an IFS A is an IFRGCS such that \(A \subseteq B \subseteq \text{cl}(A)\), where B is an IFS in an IFTS \((X, \tau)\), then B is an IFGα** CS in \((X, \tau)\).
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Proof: Let $U$ be an IFROS in $(X,\tau)$ such that $B \subseteq U$. Then $A \subseteq U$. Since $A$ is an IFRGCS and $\alpha cl(A) \subseteq cl(A)$, we have $\alpha cl(A) \subseteq cl(A) \subseteq U$. Now, $\alpha cl(B) \subseteq cl(B) \subseteq cl(A) \subseteq U$. Hence $B$ is an IFGa** CS in $(X, \tau)$.

Theorem 3.29: An IFS $A$ is an IFGa** CS in an IFTS $(X,\tau)$ if and only if $\exists(A,F)$ implies $\exists(\alpha cl(A),qF)$ for every IFRCS $F$ of $(X,\tau)$.

Proof: Necessity: Assume that $A$ is an IFGa** CS in $(X,\tau)$. Let $F$ be an IFRC and $\exists(A,F)$. Then $A \subseteq F^c$, where $F^c$ is an IFRCS in $(X,\tau)$. Then by assumption, $\exists(\alpha cl(A),F)$. Sufficiency: Let $F$ be an IFRCS in $(X,\tau)$ such that $F \subseteq U$. Then $F$ is an IFRCS in $(X,\tau)$ and $F \subseteq (U^c)^c$. By assumption, $\exists(F,qU)$ implies $\exists(\alpha cl(A),qU)$. Therefore, $\exists(\alpha cl(A),qU)$. Hence $A$ is an IFGa** CS in $(X,\tau)$.

Theorem 3.30: If $A$ is an IFROS and an IFGa** CS in $(X,\tau)$, then $A$ is an IFOCS in $(X,\tau)$.

Proof: Let $A$ be an IFROS. Since $A \subseteq A$, $\alpha cl(A) \subseteq A$. But $A \subseteq \alpha cl(A)$ always. Therefore $\alpha cl(A) = A$. Hence $A$ is an IFOCS in $(X,\tau)$.

Theorem 3.31: Every IFS in $(X,\tau)$ is an IFGa** CS if and only if IfFaOS and IfFaCS coincide.

Proof: Necessity: Suppose that every IFS in $(X,\tau)$ is an IFGa** CS. Let $U$ be an IFROS in $(X,\tau)$. Then $U$ is an IFOS and an IfFaOS by hypothesis $\alpha cl(U) \subseteq U \subseteq \alpha cl(U)$, that is $\alpha cl(U) = U$. Thus $U$ is an IFaCS in $(X,\tau)$. Hence $\exists(U,F)$ implies $\exists(\alpha cl(U),F)$. Sufficiency: Suppose that $\exists(U,F)$. Let $A \subseteq U$ and $U$ be an IFROS in $(X,\tau)$. Since every IFROS is IfFaOS, $U$ is an IFaOS in $(X,\tau)$ and therefore $\exists(\alpha cl(U),U)$. Hence $A$ is an IFGa** CS in $(X,\tau)$.

Theorem 3.32: An IFS $A$ of an IFTS $(X,\tau)$ is an IFROS and an IFGa** CS, then $A$ is an IFRCS in $(X,\tau)$.

Proof: Let $A$ be an IFROS and an IFGa** CS in $(X,\tau)$. Then $\alpha cl(A) \subseteq A$. Since $\alpha cl(A)$ is an IFaCS, we have $cl(int(\alpha cl(A))) \subseteq A$. Therefore $\alpha cl(A) \subseteq A$, since $A$ is an IFROS. Then $cl(int(A)) \subseteq cl(A) \subseteq A$. Therefore $\alpha cl(A) \subseteq A$. Since every IFROS is an IFaOS, $A$ is an IFaOS and we have $A \subseteq cl(int(A))$. Hence $A$ is an IFGa** CS in $(X,\tau)$.

Theorem 3.33: Let $A$ be an IFGa** CS in $(X,\tau)$ and $p_{(\alpha,\beta)}$ be an IFP in $X$ such that $\alpha cl(A) \subseteq cl(p_{(\alpha,\beta)})$. Then $A \alpha cl(p_{(\alpha,\beta)})$.

Proof: Assume that $A$ is an IFGa** CS in $(X,\tau)$ and $\alpha cl(A) \subseteq cl(p_{(\alpha,\beta)})$. Suppose that $\exists(A,G)$, $\exists(cl(p_{(\alpha,\beta)})G)$. Then $A \subseteq (\alpha cl(p_{(\alpha,\beta)})) \subseteq (cl(p_{(\alpha,\beta)}))$, which is a contradiction to the hypothesis. Hence $A \subseteq (\alpha cl(p_{(\alpha,\beta)})$.

4 **Intuitionistic Fuzzy Ga****: Open Sets**

Definition 4.1: An IFS $A$ of an IFTS $(X,\tau)$ is called an IFGa** OS if and only if $A^c$ is an IFGa** CS.

Theorem 4.2: Every IFOS, IFROS, IFaOS, IFGOS, IFRGOS, IFaGOS, IFGaOS is an IFGa** OS in $(X,\tau)$.

Proof: Obvious.

Example 4.3: Let $X = \{a, b\}$ and $\tau = \{0, 1\}$ be an IFTS on $X$, where $G = (x, (0.5, 0.6), (0.5, 0.4))$. Then the IFS $A = (x, (0.5, 0.6), (0.4, 0.5, 0.5))$ is an IFGa** OS but not an IFaOS, IFFaOS, IFGOS, IFaGOS in $(X,\tau)$.

Example 4.4: Let $X = \{a, b\}$ and $\tau = \{0, 1\}$ be an IFTS on $X$, where $G = (x, (0.8, 0.8), (0.2, 0.1))$. Then the IFS $A = (x, (0.1, 0.3), (0.9, 0.7))$ is an IFGa** OS but not an IFGaOS in $(X,\tau)$.
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Example 4.5: Let \( X = \{a, b, c\} \) and \( \tau = \{\emptyset, G_1, G_2, 1\} \) be an IFTS on \( X \), where \( G_1 = (X, (0, 0.4, 0.5), (0.4, 0.4, 0.4)) \) and \( G_2 = (X, (0.2, 0.3, 0.5), (0.5, 0.5, 0.5)) \). Then the IFS \( A = (X, (0.5, 0.4, 0.5), (0.4, 0.3, 0.2)) \) is an IFgα**CS but not an IFgα**OS, IFgα**CS in \( (X, \tau) \).

Theorem 4.6: An IFS \( A \) of an IFTS \( (X, \tau) \) is an IFgα**OS if and only if \( U \subseteq \text{aint}(A) \) whenever \( U \subseteq A \) and \( U \) is an IFRCS.

Proof: Necessity: Assume that \( A \) is an IFgα**OS in \( (X, \tau) \). Let \( U \) be an IFRCS such that \( U \subseteq A \). Then \( U^c \) is an IFROS and \( A^c \subseteq U^c \). Then by assumption \( A^c \) is an IFgα**CS in \( (X, \tau) \). Therefore, we have \( \text{acl}(A^c) \subseteq U^c \). Hence \( U \subseteq \text{aint}(A) \).

Sufficiency: Let \( U \) be an IFROS in \( (X, \tau) \) such that \( A^c \subseteq U \). Then \( U^c \subseteq A \) and \( U^c \) is an IFRCS. Therefore \( U \subseteq \text{aint}(A) \). Since \( U \subseteq \text{aint}(A) \), we have \( \text{aint}(A)^\circ \subseteq U \) that is \( \text{acl}(A^c) \subseteq U \). Thus \( A^c \) is an IFgα**CS. Hence \( A \) is an IFgα**CS in \( (X, \tau) \).

Remark 4.7: Intersection of two IFgα**O sets is an IFgα**O sets in \( (X, \tau) \): But the union of two IFgα**O sets need not be an IFgα**OS.

Example 4.8: Let \( X = \{a, b, c\} \) and \( \tau = \{\emptyset, G_1, G_2, G_1 \cap G_2, 1\} \) be an IFTS on \( X \), where \( G_1 = (X, (0, 0.6, 0.1), (0.1, 0.25, 0.5), (0.3, 0.7, 0.1)) \). Then the IFS \( A = (X, (0.2, 0.25, 0.3), (0.3, 0.7, 0.1)) \) and the IFS \( B = (X, (0.1, 0.3, 0.0), (0.0, 0.6, 0.3)) \) are IFgα**O sets but \( A \cap B \) is not an IFgα**O set.

Theorem 4.9: Let \( A \) be an IFS in \( (X, \tau) \). If \( B \) is an IFSOS such that \( B \subseteq A \subseteq \text{int}(\text{cl}(B)) \), then \( A \) is an IFGOS in \( (X, \tau) \).

Proof: Since \( B \) is an IFSOS, we have \( B \subseteq \text{cl}(\text{int}(B)) \). Thus, \( A \subseteq \text{int}(\text{cl}(B)) \subseteq \text{int}(\text{cl}(\text{int}(B))) = \text{int}(\text{cl}(\text{int}(B))) \subseteq \text{int}(\text{cl}(\text{int}(A))) \). This implies \( A \) is an IFOS. By Theorem 4.2, \( A \) is an IFgα**OS in \( (X, \tau) \).

Theorem 4.10: If an IFS \( A \) is an IFGOS in \( (X, \tau) \) such that \( \text{aint}(A) \subseteq B \subseteq A \), where \( B \) is an IFS in \( (X, \tau) \), then \( B \) is an IFgα**OS in \( (X, \tau) \):

Proof: Suppose that \( A \) is an IFgα**OS in \( (X, \tau) \) and \( \text{aint}(A) \subseteq B \subseteq A \). Then \( A^c \) is an IFgα**CS and \( A^c \subseteq B^c \subseteq \text{cl}(\text{aint}(A)) \), this implies \( A^c \subseteq B^c \subseteq \text{cl}(A^c) \). Then \( B^c \) is an IFgα**CS in \( (X, \tau) \), by Theorem 3.28. Hence \( B \) is an IFgα**OS in \( (X, \tau) \).

Theorem 4.11: If an IFS \( A \) is an IFRCS in \( (X, \tau) \) such that \( \text{int}(A) \subseteq B \subseteq A \), where \( B \) is an IFS in \( (X, \tau) \), then \( B \) is an IFgα**OS in \( (X, \tau) \):

Proof: Let \( A \) be an IFRCS and \( \text{int}(A) \subseteq B \subseteq A \). Then \( A^c \) is an IFGCS and \( A^c \subseteq B^c \subseteq \text{cl}(A^c) \). Then \( B^c \) is an IFgα**CS in \( (X, \tau) \), by Theorem 3.28. Hence \( B \) is an IFgα**OS in \( (X, \tau) \).

**References**